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स्वागताध्यक्ष की ओर से

आर० सी० शर्मा

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कुलपति, गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार–२४९४०४

मेरे लिये अत्यन्त प्रसन्नता का विषय है कि गुरुकुल कांगड़ी विश्वविद्यालय में "विज्ञान परिषद् आँफ इन्डिया" का अखिल भारतीय सम्मेलन दिनाँक १०-११ मार्च, १९८९ को होने जा रहा है जिसमें (१ वैदिक गणित, परम्पराएं एवं अनुप्रयोग तथा (२) आधुनिक विज्ञान एवं तकनालाजी में गणित के अनुप्रयोग पर विशेष सिम्पोजियम भी आयोजित किये जा रहे हैं। श्री स्वामी श्रद्धानन्द महाराज को यह उत्कट इच्छा रही है कि यहाँ वैदिक साहित्य, सस्कृत, धर्म, दर्शन और मानविकी के आधुनिक विषयों के साथ विज्ञान की सभी शाखा-प्रशाखाओं का उच्चस्तरीय अध्ययन हो। विज्ञान महाविद्यालय की स्थापना का उद्देश्य भी यही था। आवश्यकता यह समझी गयी कि भारतीय वैज्ञानिकों के विज्ञान सम्बन्धी शोध कार्यो का गहन अध्ययन और मूल्यांकन विश्व-विद्यालय में होना चाहिए। गुरुकुल की अपनी पहचान भारतीय दृष्टि का विकास करना और विश्व परिप्रेक्ष्य में उस दृष्टि की प्रतिष्ठा करना है। यह गणित सम्मेलन यदि इस कार्य को प्रोत्साहन करेगा तो परिसर में इस अधिवेशन के आयोजित होने की सार्थकता प्रमाणित हो सकेगी।

वेदांग ज्योतिष के साथ जुड़े गणित का भारत में गौरवपूर्ण इतिहास रहा है। आर्च और याजुष ग्रन्थों से लेकर, आर्यभट्ट, भास्कर, ब्रह्मगुप्त, आर्यभट्ट द्वितीय, श्रीधर, श्रीपति, महावीराचार्य, नारायण पण्डित, मुनीश्वर, बापूदेव शास्त्री, सुधाकर द्विवेदी जैसे ग्रन्थकारों के गणित विषयक का कार्य का आंकलन करते हुए रामानुजम जैसे गणितज्ञों का महत्व हम यदि प्रकाशित करते हैं तो वैदिक, बौद्ध और जैन परम्पराओं की विज्ञान सम्बन्धी खोजों का रोचक इतिहास हमें मिल जाता है। अंकगणित, बीजगणित तथा ज्यामिति के क्षेत्र में भारतीय गणित शास्त्र के व्यापक अनुदान का युक्तियुक्त परीक्षण और मूल्यांकन होना चाहिये।

मेरा पूर्ण विश्वास है कि जहाँ आप गणित शास्त्र के आधुनिक विचारकों की निष्पत्तियों और समस्याओं का निदान खोजेंगे वहाँ भारतीय गणित शास्त्र के सही मूल्यांकन के लिये नवयुवकों को प्रेरित भी करेंगे।

इस शुभ अवसर पर परिषद् के अध्यक्ष प्रोफेसर जे० एन० कपूर परिषद् के पदाधिकारियों एवं सम्मेलन के सभी प्रतिभागियों का विश्वविद्यालय स्वागत करता है।

सम्मेलन की निर्विघ्न सम्पन्नता के लिये मेरी हार्दिक शुभ कामनाएँ।

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A GOOD BEGINNING

By

DR. VIRENDRA ARORA

Registrar, Gurukula Kangri Vishwavidyalaya

Hardwar- 242904, U.P.

The deciphering of Brahmilipi by Subhas Kak, a renowned cryptologist and Professor of Computer Science at Lousiana State University has confirmed that the Vedic period is some what 6000-4000 B.C. Prof.Kak. through his recent publications, has come out with a surpising fact that paradoxes of modern scince and logic are well traced in Rigveda. This naturally implies that the state of science and technology during Vedic period was no less than what we have today. This fact is further supported by the exactness of measurements described in Brahmilipi.

Aryabhata' s algorithm, called Kuttaka by ancient Indian mathematicians, was meant to solve linear indeterminate equations. Such solutions are fundamental to many computer science applications, and a method called the Chinese Remainder Method is used to find them. Surprisingly Aryabhata' s method is extremely efficient compu, tationally; in fact, as efficient as recent improvements to the Chinese Remainder Method, that were developed during the last 15–20 years.

Indeed, according to Cazori, "Numerical Symbolism, the science of numbers, and algebra attained in India far greater perfection than they had previously reached in Greece..................Both the form and the spirit of the arithmetic and algebra of modern times are essentially Indian".

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It is perhaps the most appropriate time for the scientific world to understand and interpret the scientific and technological messages of our traditional system of knowledge and philosophy.

It is indeed, a matter of great pleasure that our Mathematics Department is not only organising a Symposium on Vedic Mathematics and its Traditions but also has initiated research work in our traditional mathematics and astronomy.

I thank the President and Office Bearers of Vijnana Parishad of India for rightly choosing our Vishwavidyalya as the venue for the Frist Annual Confernce whose deliberations are oraganised in the perspective of Vedic Mathematics. Prof S. C. Tyagi and Prof. S. L. Singh deserve full apreciation for organising this Conference, though the travails of successfull organising a conference is better known to them.

We are aware our lapses, and hope that the delegates will bear with us. Once again I extend a warm welcome to you all.

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हरिद्वार नगरपालिका अध्यक्ष की ओर से

पारस कुमार जैन वैद्य

हरिद्वार एक अत्यन्त प्राचीन, पवित्र तीर्थ व भारत प्रसिद्ध ऐतिहाहिक नगर है। हरिद्वार का उपनगर कनखल उससे भी पुराना स्थान है जो भगवान शिव की ससुराल है। कनखल वही स्थान है जहां मां पार्वती ने अपने पिता प्रजापति दक्ष के यज्ञ में अपने पति के अपमान पर आत्मदाह किया था। इस नगरी का पुराणों में भी वर्णन है। यह नगर "विज्ञान परिषद ऑफ इण्डिया" के अधिवेशन में पधारे वैज्ञानिकों, विद्वानों एवं शोध छात्रों का स्वागत करता है।

कनखल (हरिद्वार) हिमालय की तलहटी में स्थित होने के कारण वनौषधियों से भरा पूरा है। यही कारण है कि इस नगर में अनेक प्रसिद्ध फार्मेसियों, आयुर्वैदिक औषधियों के मिर्माण में लगी हैं जिनमें कई विदेशों में भी ख्याति अर्जित कर रहीं है। गुरुकुल कांगड़ी फार्मेसी एवं देश रक्षक औषधालय प्रमुख हैं। इस ऐतिहासिक नगरी में हरिद्वार मे प्रदेश के प्राचीनतम आयुर्वैदिक कालेज गुरुकुल कांगड़ी तथा ऋषिकुल आयुर्वेदिक कालेज गत ५० वर्षों से जनसेवा में लगे हैं। इसके साथ ही गुरुकुल आयुर्वेद महाविद्यालय भी आयुर्वेद चिकित्सा का ज्ञान मिश्रित चिकित्सा पद्धति प्रदान कर रहा है।

हरिद्वार में प्रतेक ६ वर्षों के पश्चात् अर्द्ध कुम्भ तथा १२ वर्षों के अन्तराल पर महाकुम्भ पर्व लगता है जिसमें भारत के कोने-कोने से लगभग एक करोड़ श्रद्धालु मां गंगा के पवित्र जल में स्नान कर पुण्य लाभ करते हैं, यह एक अद्भुत समारोह है। जिसमें बिना विज्ञापन व निमंत्रण के जन समूह उमड़ता है।

हरिद्वार में आने वाले श्रद्धालु भक्तों की सेवा के लिये गंगा सभा, सेवा समिति. भारत स्काउट व गाइड एवं अन्य संस्थायें एक लम्बे समय से कार्यरत है। नगरपालिका हरिद्वार नगर की बिजली, पानी व सफाई व्यवस्था का कार्य बेहतर ढंग से करने का प्रयास कर रही है।

अंत में एक बार पुनः हम हरिद्वार नगरपालिका की ओर से अधिवेशन में पधारे सभी वैज्ञानिकों और विद्वानों का हार्दिक स्वागत करते हैं; अभिनन्दन करते हैं तथा अधिवेशन की सफलता के लिये हार्दिक शुभकामना करते हैं। धन्यवाद!

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GREETINGS FROM LOCAL SECRETARY

By

S. L. Singh, G. K. University Hardwar-249404, U. P

It is a matter of great privilege for the Gurukula Kangri Vishwa-vidyalaya to be entrusted with the very pleasant and most exciting task of organising the First Session of the Vijnana Parishad of India. I consider it to be a great honour to the holy city of Hardwar more so because Hardwar is a newly born as a district.

I welcome the delegates and you all. वास्तव में,

"आना तेरा मुबारक, तशरीफ लाने वाले ।
खुशियां मना रहे हैं हम गुरुकुल वाले ।।"
"क्या मुख से विनती करु, लाज लागत मोय ।
तुम जानत अवगुण करु, कैसे भाऊं तोय ।।"
"मेरे महबूब की सूरत तुझसे मिलती जुलती है।
मुहब्बत की मुहब्बत है,इबादत की इबादत है ।।"

Our limitations to make your stay physically comfortable are enormous. We have tried our best to overcome our limitations and frankly rpeaking, depend on your generosity to forgive us for the lapses on our part.

I take opportunity to address our young dilegates. A true scientist looks beyond the boundaries of the nation he belongs to, especially the Indian scientist whose cultural heritage has been to consider the whole world as his family "वसुधैव कुटुम्बकम्". A good scientist can teach mankind to choose to live rather than to be swayed towards eradication for those, who are not scientists nor have scientific temper, are slaves of passion and are likely to misuse their power and the attainments of science. According to Kathopanishad:

"अस्तु अविज्ञानवान भवति अयुक्तेन मनसा सदा ।
तस्य इन्द्रियाणि अवश्यानि दुष्टाश्वा विवसारथे ।।"

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(One who is not endowed with science, nor is associated with it in his mind, has his senses and body not in his control like the rogue horse is not in the control of the charioteer.)

Dear friends ! the entire mankind looks towards scientists to have a promise of life.

Dear delegates ! the whole Gurukul family welcomes you once again, Hardwar welcmes you, the Ganges welcomes you, and the Nagadhiraj Himalaya welcomes you. It will be a great sense of satisfaction for me and the organizers if you carry some pleasant memories from here.

और अंत में, आप अपनी असुविधाओं और मेरी कमियों के लिए क्षमा दान दें। जैसे कि:-

"तेरे दरबार की आली निराली शान ये देखी ।
शखावत खुद तेरी गलियों में चक्कर काटती देखी ।।
फैलाया हाथ जिसने तेरे दरबार में आकर ।
तुझे देते नहीं देखा मगर झोली भरी देखी ।।"

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उद्घाटन–भाषण

आचार्य रामप्रसाद वेदालंकार
कार्यकारी कुलपति (एवं प्रो-उप-कुलपति)

माननीय श्री कपूर जी, अध्यक्ष, चन्देल जी मंत्री, अखिल भारतीय विज्ञानपरिषद, आचार्य प्रियव्रत जी वेदवाचस्पति, भूतपूर्व कुलपति, गुरुकुल कांगड़ी विश्वविद्यालय एवं मेरे सहयोगी साथियो, बाहर से पधारे हुए अभ्यागत महानुभावों तथा प्रिय छात्रो, इस युग के महामानव महर्षि दयानन्द की ब्रह्मा से लेकर जैमिनी मुनि पर्यन्त प्रतिपादित वेदानुकूल ज्ञानविज्ञान की शिक्षा के व्यापक प्रचार-प्रसार की परिकल्पना को साकार करने के लिये स्वामी श्रद्धानन्द ने हरिद्वार से ५ मील दूर गंगा के पवित्र तट पर २ मार्च १९०२ में गुरुकुल कांगड़ी की स्थापना की थी। उस समय कौन यह कल्पना कर सकता था कि इस घने जंगल में पूस के छप्परों में प्रारम्भ यह गुरुकुल आगे चलकर न केवल सरस्वती की आराधना का अपितु सामाजिक एव राष्ट्रिय जागरण का अग्रदूत बन सकेगा। अपने शैशव काल से ही गुरुकुल ने राष्ट्रीय जीवन पर अपनी अमिट छाप अंकित कर दी। यह संस्था स्वतन्त्रा आन्दोलन, पत्रकारिता, आयुर्वेद, साहित्य, विज्ञान, इतिहास, आदि का प्रेरणा केन्द्र बन गया। उस समम के सर्वश्रेष्ठ ख्याति प्राप्त नेता यहाँ पर आये और गुरुकुल के पवित्र जीवन को देखकर भाव-विभोर हो गये। इसी पावन भूमि में स्वामी श्रद्धानन्द ने बड़े आदर और प्रेम से मि. गांधी को महात्मा गांधी कहकर संबोधित किया था। गुरुकुल का यश चारों दिशाओं में फैलता गया। डा. राजेन्द्र प्रसाद, श्रीमती इन्द्रागांधी, राष्ट्रकवि दिनकर, मुंशी प्रेमचन्द जैसे अनेक राष्ट्रिय नेताओं व साहित्यकारों ने समय-२ पर यहाँ पधार कर स्तंभ को गौरवान्वित समझा। प्रारम्भ में यह गुरुकुल २२–२४ बर्ष तक शिवालिक की तराई में गंगा तट पर अपनी कीर्ति पताका फहराता रहा। गुरुकुल शिक्षा के क्षेत्र में प्रारम्भ से उन्नति के शीर्ष पर रहा है, शिक्षा जगत् में इसकी अपनी एक विशिष्ट छवि रही है, यही कारण है कि विश्वविद्यालय अनुदान आयोग की अनुकूल सस्तुति पर भारत सरकार ने १९६२ में गुरुकुल को विश्वविद्यालय की मान्यता प्रदान की। और यहाँ की सभी उपाधियाँ देश क अन्य विश्वविद्यालयों तथा केन्द्रीय एवं प्रान्तीय सरकारों द्वारा अन्य विश्व-विद्यालयों के समान मान्यता प्राप्त हो गयी। विश्वविद्यालय के अन्तर्गत वेद महा-विद्यालय कला, महाविद्यालय, विज्ञान महाविद्यालय, आयुर्वेद महाविद्यालय तथा

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देहरादून में कन्या गुरुकुल महाविद्यालय शिक्षा के व्यापक प्रचार प्रसार में लगे रहें। मध्यकाल में उत्तर प्रदेश शासन ने आयुर्वेद महाविद्यालय को अपने अधीन कर लिया। आज भी वहाँ उच्चस्तरीय अध्ययन चल रहा है। इस बिश्वविद्यालय में स्नातक परीक्षोत्तीर्ण छात्र विद्यालंकार, वेदालकार, उपाधियों से विभूषित किये जाते हैं, साथ ही वेद, दर्शन,सस्कृत, भारतीय सस्कृति एवं प्राचीनेतिहास, अंग्रेजी, हिन्दी, मनोविज्ञान आदि विषयों में द्विवर्षीय स्नातकोत्तर पाठ्यक्रम के अतिरिक्त उच्चतर अध्ययन एवं अनुसन्धान हेतु पी-एच० डी० शोधोपाधि प्राप्त करने की व्यवस्था भी यहां की गयी है। विज्ञान महाविद्यालय में गणित, भौतिकी, रसायन-शास्त्र, जन्तुविज्ञान, वनस्पति विज्ञान आदि की उच्चस्तरीय शिक्षा के साथ कम्प्यूटर साइंस की शिक्षा भी प्रारम्भ हो चुकी है। वैदिक प्रयोगशाला, योग डिप्लोमा, एवं कर्मकाण्ड डिप्लोमा इस विश्व-विद्यालय की नवीनतम उपलब्धियाँ हैं। यहाँ के प्राध्यापक एवं छात्र वैदिक प्रयोग-शाला में अनुसन्धान कार्य में लगे रहते हैं। गुरुकुल पत्रिका, प्रहलाद् वैदिक-पथ आदि विभिन्न शोध-पत्रिकाएं दूर-दूर तक ज्ञान-विज्ञान के प्रचार में संलग्न हैं। समय-२पर शोध-सगोष्ठियों का समायोजन करके उनमें राष्ट्रिय एवं अन्तर्राष्ट्रिय ख्याति प्राप्त विद्वानों को कुलभूमि में आमन्त्रित किया जाता है। जिससे प्राचीन एवं अर्वाचीन के मधुर समन्वय मे राष्ट्रीय-जीवन को महिमा-मण्डित करने का अखण्ड प्रयास निरंतर चलता रहता है। इस सस्था मे कार्यरत अधिकारियों एवं शिक्षकों के मस्तिष्क में सदैव राष्ट्र कल्याणकारी शिव-सकल्प जागते रहते हैं। यहाँ के शिक्षक सतत इस चिन्तन में संलग्न रहते हैं कि किस प्रकार समाज के सभी वर्ग इस तीर्थ में स्नान करते रहें, इसी सद्भावना के साथ आज भारतीय विज्ञान परिषद् द्वारा हमारे इस गुरुकुल विश्वविद्यालय में समायोजित परिषद् का प्रथम वार्षिक अधिवेशन तथा Vedic Mathematics संगोष्ठी के शुभ अवसर पर आप सबका पुनः हार्दिक स्वागत एवं अभिनन्दन करता हुआ अधिवेशनतथा सगोष्ठी का शुभारम्भ करता हूं।

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VIJNANA PARISHAD OF INDIA
FIRST ANNUAL CONFERENCE , 1989
GURUKUL KANGRI VISHWAVIDYALAYA , HARDWAR
PRESIDENTIAL ADDRESS

By

Professor J.N. Kapur, Ph.D. ; F.N.A.Sc. ; F.A.SC. ; F.N.A.

At the out set , I must express my sincere thanks to all members of"Vijnana Parishad of India'' for the honour they have done me by asking me to be the first President of the Parishad. I must also thank the authorities of Gurukul Kangri Vishwavidyalaya for inviting the Parishad to hold its First Annual Conference in this holy city of Hardwar.

Along with this conference , we shall also be holding a National Symposium on "VEDIC MATHEMATICS, TRADITIONS AND APPLICATIONS". This is appropriate, since this university has many great scholars of Sanskrit and Vedas and as such this Symposium should provide a unique opportunity for meaningful interaction between mathematicians and Vedic scholars , so that we can know more about the truth regarding the origins of the great traditions of Mathematics in India,

India was certainly a world leader in Mathematics till at least the twelth century and the foundations of this greatness were laid by the great rishies and Vedic scholars who thought about great truths about numbers and geometrical forms in deep forests and in the high ranges of the Himalyas and in holy places like Hardwar . These rishies have not only left us mathematics of which we can be proud, they have also left us great traditions of dedication to learning and scholarship. They were great worshipers of Sarswati and the mathematicians of modern India can draw a great inspiration fron these

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great seers who pursued mathematical studies vigrously and unrelentingly for the sheer joy it gave them.
Vedic Mathematics means Mathematics in the Vedas. However recently Vedic Mathematics has come to be associated with the mathematics contained in the book with the title written by Jagatguru Shankaracharyaji Maharaj.

The sixteen sutras or aphorisms carrying out mathematical computations given in thish book have caught the imagination of hundreds of thousands of people in India and abroad. There has been a great enthusiasm for these formulae among the laymen everywhere.

While it is good to be enthusiastic, we should not be swept off our feet by it. We should remeber some basic facts:

(1) The book "Vedic Mathematics" does not deal with "Mathematics in the Vedas" for which there are many other books. It deals with only sixteen Ganita sutras which Swamiji stated were contained in an Appendix of Atharvaveda.

(ii) This Appendix is not found in any of the various of the Vedas availble with us inspite of deliberate search for these. I am sure the great scholars of the Vedas in this university will be able to tell us if there is anything resembling these in the Vedas.

(iii) When Professor K.S. Shukla, a very distinguished scholar of ancient Indian Mathematics took a copy of Atharveda to Swamiji personally and asked him to show where in it, these sutras were contained, he said: These are contained in a copy of Atharva Veda which I have but he did not produce this copy inspite of Dr. Shukla's request.

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(iv) In an Address to university students in California, Swamiji stated that these formulae were in one place in the body of the Atharvaveda under the title of Ganita sutras' and his contribution was to interpret these properly with the help of ancient lexigraphies and by deep mediation in forests. However the term 'Ganita' was not used in the Vadas and was used only later by Jain scholars.

(v) Swamiji's desciple Manjula Trivedi stated that these sutras where not contained in the vedas as such, but were constructed from material's scattered in the vedas here and there. This Contradicts Swamiji's own statement.

(vi) As Professor Shukla has demonstrated, most of the ideas contained in these sutras were developed as late as eighteenth century and are not all based on ideas otherwise known to be contained in vedas. Swamiji was familiar with all this mathematics because he had done M.A. in mathematics. We do not expect a great charm the mathematcal ideas contained in the rest of vedas and ideas contained in the sixteen sutras also said to be contined in the vedas.

(vii) The language of these sutras is quite different from vedas Sanskrit. Again the vedic scholars here can throw light on the aspect.

(viii) It is said that whether 'Vedic Mathematics' belongs to Vedas or not may be controversial, but that is good mathematics can not be denied. It is interesting mathematics,it is enjoyable mathematics, it is recreational mathematics but it is elementary mathematics.

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(ix) These sutras simplify calculations and simplify considerably for some problems, for other problems the calculations may even become longer. As such these cannot be used to replace our standard methods of calculation.

(x) Though in some situations these can give very quick answers, again these cannot compete with calculators in finding 89^{89}. *In* 89, sin 89 etc.

(xi) There is no doubt that both children and adults enjoy these short cuts tremendously. These short cuts are obtained by a deep understanding of the place-value system. What these demonstrate is that mathematics can be best understood by understanding its structure.

(xii) What this book demonstrates is that mathematics can greatly enjoyable if it is presented properly. Swamiji was a great master in explaining mathematical matters lucidly, clearly and in a fascinating manner.

(xiii) This book also demonstrates the great intellectual joy which mathematics alone can give. This book represents a very very small part of mathematics. If this small part can give so much enjoyment, one can expect wonderful thrils from mathematics, provided it is explained and understand properly.

(xiv) These opinions about the book 'Vedic Mathematics' are my own and do not represent the views of the parishad. In fact the Parisad has organised this symposium to know the Truth and Truth nomes when there is a free expression of opinions by all.

One object of the Parishad is to make possible the 'Enjoyment of Mathematics' by millions of our countrymen. Mathematics can

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prosper only if millions of people enjoy it and love it. Today the situation is just the opposite. Due to disastrous teaching of mathematics in our schools, millions have developed a dislike for mathematics they suffer from mathphobia. They respect it but they believe that mathematics is not their cup of tea.

We have to revers this state of affairs. We do not want just respect for mathematics, we want love for mathematics we are extremely greatful to Swamiji for demonstrating that mathematics can be enjoyed. We want hundreds of missionaries like Swamiji to go out and spread the message of fascination of Mathematics. I have myself written extensively on this subject.. Two hundred of my articles are included in the eight volumes of my book, Fascinating World of Mathematical Sciences, published by Mathematical Sciences Trust Society, New Delhi

Tte major objective of the Parishad is to work for Propagating Applications of Mathematics in all walks of life. We would like to promote Applications of Mathematics in curricula of primary schools, secondary schools,undergraduate colleges and postgraduate departments.

Our students are not learning sufficient applications of mathematics to motivate them towards learning of mathematics. Accordingly a few years ago UNESCO commissioned me to write a book on these applications, using only secondary school mathematics, for the use of future teachers of Mathematics under preservice training in training colleges, I accordingly wrote the book' Insight into Mathematical Modelling' which was published by Indian National Science Academy. This book gives about one hundred applications of mathematics in astronomy geography, physics, physiology, management finance etc.

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At the undergraduate level, we want to have a compulsory course in Statics, Dynamics and Hydrostatics and this used to account for one-third of the undergraduate course. Over the years this course has been diluted or entirely replaced by courses on pure mathematics, so much so that a student can get B.Sc. degree without kdowing ane single application of mathematics. This is similar to the situation that occured eralier in other parts of world. To remedy this, undergraduate courses in mathematical modelling were introduced there. These are are also being introduced in our country and I have written a book on 'Mathematical Modelling' for this purpose which has been published by Wiley Eastern, New Delhi and John Wiley, New York. This gives aqplictions of ordinary differential equations, difference equations, pratal differential equations, integrodifferential equations, graph theory, optimization techniques etc. to problems in physics, economic, biology, astronomy, environment control, population control, epidemics, genetics etc.

At the postgraduate level, courses on spherical Astronomy, Hydrodynamics and Electromagnetic Theory used to be popular. Later new courses in mathematical Statistics and operations research was introduced. More recently courses on Biomathematics have been introduced. I have written a book on 'Mathematical Models in Biology and Medicine' and another on 'Maximum Entropy Models in Science and Engineering' which can be used at this level.

Our main problems however are at the research level. Mathematics in India has not played the role it should have played in helping in solving problems of national life viz. Problems which arise in CSIR, DRDO, ISRO and other laboratories, problems which our industry faces, problems which large scale use of computers has

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introduced, problems of pollution and population control, problems of modern engineering like pattern recognition, artificial intelligence, robotics etc. , problems of atmospheric and ocean sciences and so on.

Our universities have failed to poduce real industrial and applied mathematicians. We have prodused enough Ph.D.'s in academic applied mathematics. However we have produced very little real life applied mathematics.

The goals of the Parishad for the next few years are clear.

(i) To work for popularisation of mathematics among students and public.

(ii) To work for introduction of more 'applications of mathematics in curricula at all levels.

(iii) To work for greater interaction of mathematics and users of mathematicis for the solution of those national problems which require the use of mathematics.

(iv) To intensify training and research in Industrial and applied mathematics.

If we can use mathematics maningfully, we can save hundreds of crores of rupees for the country.

There are nany challenges for mathematieians in the modern world. I am sure that members of the Parishad will accept the challenge of placing India on the world map of Industrial and Applied Mathematics.

The Parishad seeks the co-operation of all of you in this task.

I hope we shall have very meningful and exiting discussion in the conference and the conference will give us goals for which we shall work in the next years ahead.

Jai Ganit Jai Hind.

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introduction, problems of pollution and population control, problems of modern engineering like pattern recognition, artificial intelligence, robotics etc., problems of atmospheric and ocean sciences and space.

Our universities have failed to produce real industrial and applied mathematicians. We have produced enough Ph.D's in academic applied mathematics. However we have produced very little real life applied mathematics.

The goals of the Parishad for the next few years are clear:

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(iii) To work for greater interaction of mathematicians and users of mathematics for the solution of large national problems which require the use of mathematics.

(iv) To intensify training and research in industrial and applied mathematics.

If we can use mathematics meaningfully, we can save hundreds of [illegible] rupees for the country.

There are many challenges for mathematicians in the modern world. I am sure that members of the Parishad will accept the challenge of placing India on the world map of industrial and applied Mathematics.

The Parishad seeks the cooperation of all of you in this task. I hope we shall have very meaningful and useful discussions in this conference and the conference will give us goals for which we shall work in the next years ahead.

[illegible]

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Proc. VPI., Vol. 1, 1989

FIRST ANNUAL CONFERENCE OF VIJNANA PARISHAD OF INDIA

GURUKUL KANGRI UNIVERSITY, HARDWAR, March 10-11, 1989

AN INTRODUCTION OF "VIJNANA PARISHAD OF INDIA"

(Society for Applications of Mathematics)

By

R. C. Singh Chandel

Secretary, Vijnana Parishad of India, D. V. Postgraduate College Orai-285001, U. P., India

Honourable Vice-Chancellor, G. K. University Hardwar, Principal Tyagiji, President Prosessor Kapur, Local Secretary Professor S. L. Singh, Delegates and Guests:

I am personally grateful to this assembly which has given me an oportunity to speak and introduce with the "VIJNANA PARISHAD OF INDIA", which is a society dedicated to Applications of Mathematics in all fields of Science and Technology and as such it has a crucial role to play because it is only the society in India explicitely devoted to this goal.

In 1970, Dr. B B. Lal the Ex-Principal, D. V. Postgraduate College., Orai 285001, U. P. desired of his science teachers to start a publication of a Scientific Research Journal from the College, containing a treasure of quotable references for the help and guidance of research scholars and worth preservable by the Libraries. Responding to his inspiration, Dr. R.C. Singh Chandel of Mathematics Department came forward to fulfil his dream and to facilitate the research activities and encourage the young researchers by taking of the publication of Mathematical Research Journal "JÑĀNĀBHA" ज्ञानाभ

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(Ray of Knowledge) published by "VIJNANA PARISHAD" of D. V. Postgaduate College, Orai. It was started in 1970-71, with generous co-operation of various experts as active members of its Editorial Board, renowned Professor J. N. Kapur as its Chief Advisor, Dr. B. B. Lal as Patron and R. C. Singh Chandel as Editor.

As soon as the Journal "JÑĀNĀBHA" saw the light of the day in 1971 it attracted the attention of the scholars of the World and crossed the boundary os the Country in no time. As it could not be kept limited to the College, the "VIJNANA PARISHAD'. of D. V. Postgraduate Colleg, Orai adopted slightly modified name "VIJNANA PARISHAD OF INDIA" in place of "VIJNANA PARISHAD", indicating an extension of its field to other countries, and besides, having an inependent status of its own free from the control of an institution. Here we can not forget the good name of Professor H. M. Srivastava (University of Victoria, Canada), who in 1972 was invited to work as a Foreign Secretary of the PARISHAD and who gladly agreed to work as one of the Editors. Some foreign experts of different fields of Mathematics and other Sciences were also included in the Editorial Board,

Since the begining of 1972 "VIJNANA PARISHAD OF INDIA" has Reciprocity Agreement with "AMERICAN MATHEMATICAL SOCIETY". The Parishad is one of the following four Indian Societies which have mutual "RECIPROCITY AGREEMENT" with "AMERICAN MATHEMATICAL SOCIETY": 1. Allahabad Mathematical Society 2. Calcutta Mathematical Society 3. Indian Mathematical Society and 4. Vijnana Parishad of India.

The Mathematical content of the Journal "JÑĀNĀBHA" of "VIJNANA PARISHAD OF INDIA" is indexed in the American

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Biweekly "CURRENT MATHMATICAL PUBLICATIONS" and is reviewed among oshers, by Mathematical Reviews (U. S. A.), Zetralblatt fur Mathematik (W. Germany) and Applied Mechanics Reviews (U. S. A.).

Upto 1985, the Parishad was an unregistered body and its aim was only to publish regularly the Research Journal "JÑĀNĀBHA". On 24th August, 1986 under the Presidentship of Professor J. N. Kapur a meeting was held to I. I. T. Kanpur and it was decided to get it registered. Consequently it was got registered on 4th July 1988 with D. V. Postgraduate College, Orai-285001, U. P. as its registered office having the following aims and activities:

AMIS OF THE PARISHAD

To work for the development of Mathematical Sciences in India. To encourage the interaction of Mathcmasics with Engineering, Industrial Medical, Social, Physical, Biological and Computer Sciences, and specially

1. To work for the popularisation of Mathematics among students and public.
2. To work for introduction of the more "Applications of Mathematics" in curricula at all levels.
3. To work for greater interaction of Mathematics and users of Mathematics for the solutions of those national proplems which require the use of Mathematics.
4. To intensify training and research in Industrial and Applied Mathematics.

ACTIVITIES

1. To publish Research Journals and Monographs in Mathematical Sciences.

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2. To publish Journals for popularising Mathematical Sciences and their teaching techniques.
3. To hold conferences, Symposia, Exhibitions and Fairs for the development of Mathematical Sciences.
4. To establish Libraries for Research Scholars.
5 To encourage Mathematics Education
6. To publish abvance books.
7. To encourage the interdiciplinary activities
8. To establish Local Chapters of the Parishad in different Universities.
9. To award prizes to the talented students
10. To award prizes and honours to the teachers and researchers of India for their unique work of eminence.
11. To take other steps considered necessary to promote the aims of the Parishad.

At present Parishad has 65 Life Members including 22 Foreign members. Parishad gets 18 Journals in Exchange of JÑĀNĀBHA including 8 Foreign Journals.

It could not be out of place to mention here that since the very inception of the first publication of the Journal JÑĀNĀBHA or in other words since last 19 years we are carrying on our mission by our own resources and have received no financial aid from the Government or from any other agency. Further it is a matter of great satisfaction and appreciation of the Parishad that the Journal is being published regularly with poor financial resources and only scanty help from voracious readers, researchers and authors. Every one here can realize the difficulties which our Parishad has been facing for the publication of the Journal and fulfilling the aims of the Parishad. No organization or Association can run without monetory funds. Our Mathmatical progress may too hemper for want of necessary funds. As such the

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Parishad has to look to the perpetual resources from which we may continue to fulfil the aims of the Parishad. In case we happen stand care free in the finacial domain for regular publication to the Journal and execution of other activities aimed at by the Parishad, we can carve out an important place for it in the Mathematical World, and the torch of our mission may give light to other countries too.

Here we are holding First Annual Conference of "VIJNANA PARISHAD OF INDIA" under the auspices of Gurukul Kangri University Hardwar, partially sponsored by DRDO, New Delhi. As a part of the Conference, we are also organising a NATIONAL SYMPOSIUM on " VEDIC MATHEMATICS, APLICATIONS AND TRADITIONS" for which we are very much thankful to the Honourable Vice-Chancellor Shri R. C. Sharma, Local Secretary Professor S. L. Singh, Principal Tyagiji and the Registrar Dr. Arora. This is only due to their keen interest that we could get together here at a very short notice.

I thank you once again the brothers and sisters assembled here from different parts of the country and welcome them, and also seek their full co-operation and good wishes for the success of the conference.

Jai Parishad

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Proc. VPI., Vol.1, 1989

REPORT ON FIRST ANNUAL CONFERENCE OF "VIJNANA PARISHAD OF INDIA"

(March 10-11, 1989)

VENUE: Gurukul Kangri University, Hardwar, U. P.

By

R. C. Singh Chandel

Secretary, Vijnana Parishad of India, D. V. Postgraduate College
Orai-285001, U. P., India

First Annual Conference of "VIJNANA PARISHAD OF INDIA was held at Hardwar under the auspices of Gurukul Kangri University Hardwar on March 10-11, 1989 partially supported by DRDO, New Delhi. The Conference was presided by Professor J. N. Kapur (Delhi) and inaugrated by the Vice-Chancellor, Gurukul Kangri University, Hardwar. From all parts of the country, 67 delegates participated in the conference out of which the following were the distinguished persons of Mathematics:

Professor J.N. Kapur (Delhi), S. N.Pandey (Roorkee), Chandra Mohan (Roorkee), Krishna Lal (Varanasi), R. N. Kaul (Delhi). Aruna Kapoor (Delhi), V. P. Saxena (Gwalior), S. P. Arya (Delhi), Narendra Puri (Roorkee), R. S. Lal (Siwan), J. M. C. Joshi (Nainital), V. N. Sharma (Rampur), R. C. Singh Chandel (Orai), S. Lal (Agra) R. S. Tripathi (Gorakhpur) and P. R. Subramanian (Madras).

During the Conference of two days, 15 invited talks were given while 35 papers were presented.

In his inaugral speach, the Vice-Chancellor of Gurukul Kangri University, Hardwar exhorted that keeping this University venue of the Conference, which was founded for the propagation of Vedas the Conference will be quite successful if in output we are able to use

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our Vedas in the progress of Modern Science and Technology through the "VIJNANA PARISHAD OF INDIA".

The Programme included Presidential address (General as well as Technical) by Professor J. N. Kupur besides other key note addresses, invited talks and paper reading sessions: specially one for National Symposium on "MATHEMATICS, APPLICATIONS AND TRADITIONS" and other for "APPLICATIONS OF MATHEMATICS IN MODERN SCIENCE AND TECHNOLOGY". The focal theme of the Conference was to discuss VEDIC MATHEMATICS and to present newer areas of use of Mathematics in Engineering, Physical, Biological, Social, Medical and Computer Sciences.

"Vedic Mathematics" was hot topic of the Conference. Dr. Naredra Puri (Roorkee) in his talk on "OVERVIEW OF VEDIC MATHEMATICS" presented some "Sutras" in Sanskirat and make their applieations in solving some particular problems of Arithmetic and claimed that sutras so used were picked up from Vedas. On the other hand some speakers like Professor Kapur and Dr. Shashi Prabha Arya expressed the view that the Mathematics particularly so called "VEDIC MATHEMATICS" was not present in any of the Vedas. This fact can be visualized by the Sanskirat language itself used in "VEDIC MATHEMATICS" written by Shankaracharya. This is different from the Sanskirat used in Vedas. Then the supporters claimed that it was present in those portions of the Vedas which have since lost. Then it was the final consensus that the so called "VEDIC MATHEMATICS" by Shankracharya is not "VEDIC MATHEMATICS". Actually, it is Ancient Mathematics which may be called "HINDU MATHEMATICS". Seeing that these sutras no doubt we may make fast calculations of some particular problems of Arithmetic very easily, it was recommended to make further researches to get applications of these

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sutras in Higher Mathematics and in Modern Science and Technology so that India may bring again her name on the international map and can guide again the World as a *Jagadguru* in the field of Science and Technology

Professor J. N. Kapur in his Presidential Address stated that "Vijnana Parishad of India" is a society dedicated to Applications of Mathematics in all fields of Science and Technology and as such it has a crucial role to play because it is the only society in India explicitely devoted to this goal. He pointed out the great job opportunities for properly trained mathematical scientists. He wanted all mathematicians to come forward to play a leadership role in the computer revolution. He wanted all Mathematics Departments to be epuipped with computer laboratories and all curricula to take into account modern applictions and computers.

In the Symposium on "VEDIC MATHEMATICS", he expressed his views that "Vedic Mathematics" by Jagadguru Shankaracharya did not deal with mathematics of the Vedic times, its level is that of Intermediate level, but it can have a great role as enrichment material to remove mathematical anxiety from the students, of course it cannot compete with computers which are million times faster.

In his velidictory address. he invited every one to have intense love for the fascinating and exciting subject of Mathematics and to work hard to learn and teach new areas of applications of Mathematics.

Professor Kapur also inspired the delegates coming from all parts of the country to execute the recommendations of the Conference in their Institutions and to further the atmosphere of researches of mathematics and their Applications in Science and Technology.

The conference came to a close with vote of thanks to the University authorities particularly the learned Vice-Chancellor Shri R. C. Sharma "Lions" of Rishikesh, Local organising Secretary Professor S. L. Singh Conveners and members of different Committees making the Conference a success.

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Income and Expenditure Account for the Year Ended 31st March, 1974

Income	Amount Rs.	Excpenditure	Amount Rs.
Opening Balance	3156.70	Postage	44.00
By Subscription and Publication Charges	1289.58		
		Total	44.00
		Closing Balance	4402.28
	4446.28		4446.28

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1975

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	4402.28	Postage	513.95
By Subscription and Publication Charges	3975.22	Publication	4875.09
		T.A.	260.90
		Miscellaneous	134.20
		Total	5784.14
		Closing Balance	2593.36
	8377.50		8377.50

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

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Inceme and Expenditure Account for the Year Ended 31st March, 1976

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	2593.36	Postage	567.75
By Subscription and Publication Charges	8363.20	Pblication	3547.19
		T. A.	31.96
		Miscellaneous	31.80
		Total	4178.70
		Closing Balance	6777.86
	10956.56		10956.56

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1977

Incom e	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	6777.86	Postage	361.60
By Subscription and Pubtication Charges	4463.31		
		Closing Balance	10879.57
	11241.17		11241.17

(Sd.) B. B. Lal
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Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

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Income and Expenditure Account for the Year Ended 31st March, 1978

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	10879.57	Postage	581.60
By Subscription and publication Charges	1934.55	Publication	5420.00
		T. A.	24.66
		Total	6026.26
		Closing Balance	6787.86
	12814.12		12814.12

(Sd.) B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1979

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	6787.86	Postage	451.05
		Publication	1000.00
By Subscription and Publication Charges	1170.99	T. A.	60.38
		Miscellaneous	28.10
		Total	1539.53
		Closing Balance	6419.32
	7958.85		7958.85

(Sd.) Dr. B. B. Lal
Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R C Singh Chandel
Secretary and Treasurer

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Income and Expenditure Account of the Year Ended 31st March, 1980

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	6419.32	Postage	97.30
By Subscription and	4478.30	Publication	5500.00
Publication Charges etc.		T. A.	80.37
		Total	5677.67
		Closing Balance	5219.95
	10897.62		10897.62

(Sd.) B. B. Lal
Patron and Pirncipal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

Income and Expenditure Account of the Year Ended 31st March, 1981

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	5219.95	Postage	584.65
By Subscription and	4456.14	Publication	1000.00
Pubtication Charges etc.		S. A.	110.76
		Total	1695.41
		Closing Balance	7980.68
	9676.09		9676.09

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Patron and Principal

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary and Treasurer

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Income and Expenditure Account for the Year Ended 31st March, 1982

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	7980.68	Postage	573.70
By Subscription and Publication Charges	2501.03	Publication	2000.00
		T. A.	80.56
		Miscellaneous	15.00
		Total	2669.26
		Closing Balance	7812.45
	10481.71		10481.71

(Sd.) B. B. Lal, President (Sd.) Tilak Singh, Society Auditor (Sd.) R. C. Singh Chandel, Secretary-Treasurer

Income and Expenditure Account for the Year Ended 31st March, 1983

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	7812.45	Postage	550.65
By Subscription, Publication Charges and loan of Rs. 725.00 given by R C.S. Chandel on 1-1-83	9333.89	Publication	15078.15
		Miscellaneous (Refund of loan)	725.00
		Total	16353.80
		Closing Balance	792.54
	17146.34		17146.34

(Sd.) B. B. Lal, President (Sd.) Tilak Singh, Society Auditor (Sd.) R. C. Singh Chandel, Secretary-Treasurer

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Income and Expenditure Account for the Year Ended 31st March, 1984

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	792.54	Postage	1682.25
By Subscription and Publication Charges	9057.10	Publication	5845.00
		Stationery	72.50
		Miscellaneous	234 40
		Total	7834.15
		Closing Balance	2015.49
	9849.64		9849.64

(Sd.) B. B. Lal
President

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary–Treasurer

Income and Expenditure Account of the Year Ended 31st March, 1985

Income	Amount Rs.	Expenditure	Amount Rs.
Opennig Balance	2015.49	Postage	891.75
By Subscription, Pubtication Charges and Loan of Rs . 1500.00 given by R. C .S Chandel	8215.78	Publication	7334.00
		Total	8225.75
		Closing Balance	2005.52
	10231.27		10231.27

(Sd.) B. B. Lal
President

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary–Treasurer

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Income and Expenditure Account for the Year Ended 31st March, 1986

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	2005.52	Postage	1311.00
By Subscription and Publication Charges	14361.17	Publication	9800.00
		Loan refund	1500.00
		Miscellaneous	5.00
		Total	12616.00
		Closing Balance	3750.69
	16366.69		16366.69

(Sd.) J. N. Kapur, President (Sd.) Tilak Singh, Society Auditor (Sd.) R. C. Singh Chandel, Secretary–Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

Income and Expenditure Account for the Year Ended 31st March, 1987

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	3750.69	Postage	1449.90
By Subscription and Publication Charges	11300.23	Publicaton	13000.00
		Total	14449.90
		Closing Balance	601.02
	15050.92		15050.92

(Sd.) J. N. Kapur, President (Sd.) Tilak Singh, Society Auditor (Sd.) R. C. Singh Chandel, Secretary –Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

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Income and Expenditure Account for the Year Ended 31st March, 1988

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	601.02	Publication	5011.00
By Subscription and Publication Charges	4923.82		
		Closing Balance	513.84
	5524.84		5524.84

(Sd.) J. N. Kapur, President — (Sd.) Tilak Singh, Society Auditor — (Sd.) R. C. Singh Chandel, Secretary-Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

Income and Expenditure Account for the Year Ended 31st March, 1989

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	513.84	Postage	4930.90
By Subscription and Publication Charges	23147.51	Publication	5988.75
		Conference	8326.00
		Stationery	218.00
		Miscellaneous	15.00
		Total	19478.65
		Closing Balance	4182.70
	23661.35		23661.35

(Sd.) J. N. Kapur, President — (Sd.) Tilak Singh, Society Auditor — (Sd.) R. C. Singh Chandel, Secretary -Treasurer

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LIST OF PARTICIPANTS WITH THE TITLES OF THEIR TALKS/PAPERS PRESENTED IN THE FIRST ANNUAL CONFERENCE OF "VIJNANA PARISHAD OF INDIA" HELD AT HARDWAR ON MARCH 10-11, 1989.

1. Professor J. N. Kapur, C-766, New Friends Colony, New Delhi
   President, Vijnana Parishad of India,
   Presidential Address, General and Technical; and Valedictory Address.
2. Dr. R. C. Singh Chandel, D. V. Postgraduate, College, Orai-285001, U. P.
   Secretary, Vijnana Parishad of India,
   Secretorial Report and a talk on
   "Binomial analogues of addition theorems of Srivastava, Lavoie and Tremblay and their applications"
3. Dr. Narendra Puri, Department of Civil Engineering, Roorkee University, Roorkee, U. P.
   "Overview of Vedic Mathematics"
4. Professor S. N. Pandey, Roorkee Universiry, Roorkee, U. P.
   "Gravitational significance of some hyper surfaces imbeded in a 5-flat".
5. Professor, R. N. kaul, Delhi University, Delhi
   "Steiner's problems and mathematical programming"
6. Professor Chandra Mohan, Roorkee University, Rookee, U. P.
   "Computational aspects of non-linear optimization problems and their practical applications".
7. Professor V. P. Saxena, Jiwaji University, Gwalior, M. P.,
   "Modern Trends in Bio-Mathematics"
8. Professor K. Lal, B. H. U., Varanasi, U. P.
   "Mathematics and its applications to Technology"
9. Professor Aruna Kapoor, Jamia Millia Islamia, Delhi "Applicaiions of Mathematics to certain problems of Duct Acoustics"
10. Professor J. M. C. Joshi, Kumayun University, Nainital, U. P.
    "S. R. RAMANUJAN - A Historical Perspective"
11. Dr. Shashi Prabha Arya, Maitrayi College, New Delhi
    "Mathematics in Vedic Literature"

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12. Professor R. S. Lal, D. A. V. College, Siwan, Bihar
"A novel method of factorization by Vedic Mathematics"
13. Dr. R. S. Tripathi, Gorakhpur University, Gorakhpur, "Developments of numbers in Vedic Age and India-Russia relations"
14. Dr. S. Lal, Agra University, Agra "Some examples in Algebra"
15. Dr. V. N. Sharma, Govt. Raja P. G. College, Rampur-244901,
"The factor 10^{27} in Hindu numeration and indeterminancy principle"
16. Professor P. R. Subramanyan, Department of Nuclear Physics, University of Madras. Madras-600025
"A Physicist's way of checking the corrctness of Mathematical equations"
17. Dr. (Mrs.) Bandana Ghosh, D. C. H. College, Dakhin Barsat, South 24 Parganas (W. B.)
"Unification of trilateral generating functions"
18. Mr. R. S. Chandel, Hoshangabad College, Hoshhangabad (M. P.)
"A common fixed point theorem of computing mappings in Saks space"
19. Mr. V. S. Dubey, Gorakhpur University, Gorakhpur,
"Decomposition of curvature tensor in R-recurrent finsler space with non-symmetric connection"
20. Dr. Ashok Ganguly, G. S. I. T. S., Indore-452001, M. P.
"Banding Moments and Shear in Bernoulie"
21. Dr. C. B. Gupta, B. S. A. College, Mathura, U. P.
"Arithmatic and Geometry in Vedas"
22. Miss Archana Lala, Bundelkhand University, Jhansi, U. P.
'Inversion of an integral involving a generalised Hermite function"
23. Mr. Indrajeet Singh, A. P. S. University, Rewa, M. P.
"Some integrals involving the product of the double hypergeometric functions and multivariable H-function"
24. Dr. R.K. Prasad, Maharaja College, Arrah (Bihar)
"On approximate solutions of multivalued mappings in ordered Banach space"
25. Dr. S. K. Ghosh, Fellow Scientist. C. R. R. I., New Delhi
"On the mixed boundary value problem in the theory of a asymmetric Elasticity'.
26. Mr. Hemant Kumar, Telephone Exchange, Orai-285001,[U. P.
"A special class of polynomials."
27. Mr. Arvind Kumar Dixit, Govt. Higher Secondary School, Mobai, Tikamgarh, M. P.
'Vedic Mathematics"

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28. Dr. H. C. Agrawal Bundelkhand College, Jhansi, U. P., "Some theorems on generating functions"
29. Dr. U. N. Singh, Magadh University, Bodh Gaya, Bihar "On approximate solutions of multivalued mappings in ordered Banach spaces"
30. Professor R. D. Agrawal, S. A. Technological Institute, Vidisha, M.P. "An application of generalized Mehler Fock transformation"
31. Mr. Himanshu Pandey, Department of Mathematics, Univereity of Gorakhpur, Gorakhpur, U. P. "Heterogeneous population growth model"
32 Dr Sambhu. Nath Singh, Maharaja College, Arrah, Dist. Bhojpur, Bihar. "On generating relations of polynomial set $\{U^*_n(x,y)\}$
33. Dr Ramji Singh, Maharaja College, Arrah, Dist Bhojpur, Bihar "Some integral represzentions of generalized polynomial set $\{M^*_n(x\ y)\}$
34. Dr B. N. Dwivedi, Atarra College, Atarra, Dist. Banda, U P. "Hydro-magnetic stability of parallel shear flows between two parallel plates"
35. Dr. Hari Kishan, Atarra College, Atarra, Dist. Banda, U.P.
    (a) Area of polygon
    (b) Generalization of Feramat's last problem
36. Dr T. Dixit, Govt. Postgraduate College, Rishikesh, "Accelaration covariance in M. H. D. terbulent dustry flow"
37 Dr. R Mall Govt. P. G. College, Rrshikesh, 'A no.e on MHD flow of an incompressible conducting fluid over an inclined non-conducting impervious boundary"
38 Dr. Kusum Shanker, Fellow Computer Centre, Central Building Research Institute, Roorkee-274667, U. P. "A system Analysis approach to Town Planning: A case study of National Capital Region, Delhi"
39. Rekha Mehndiratta, Cottage No. 4, Old Brockhurst, Shimla-9. "Various fixed point theorems"
40. Ms. Pragati Goel, Research Scholar, Roorkee University, Roorkee, U.P. "A two phase Mathematical model and Statical Analysis for drag reduction in lamlnar flow"

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41. Dr. J. S. Verma Praveen Tech. Institute, Majra, Dehradun
"Geometry in Vedic Period"
42. Professor S. L. Singh, G. K. University, Hardwar, U. P., Local Secretary of the Conference
43. Professor S. C. Tyagi Principal, Science College, G. K. University Hardwar
"Welcome"
44. Dr. H. L. Gulati, G. K. University, Hardwar, U. P.
"On Job-block open shop scheduling problems"
45. Dr. P. P. Pathak, G. K. University, Hardwar, U. P.
"Decimal System in Time scale"
46. Mr. M. P. Singh c/o-Dr. S. L. Singh, G. K. University, Hardwar.
"Vibration analysis of a non-uniform circular plate subjeced to one dimensional steady state temperature distribution
47. Mr. Vijayendra Kumar, G. K University, Hardwar
"२- दूरीक समष्टि में कुछ स्थिर बिंदु प्रयोग"
48. Mr. U. C. Gairola, G. K. University, Hardwar, U. P.
"A general coincidence theorem"
49. Mr. Ramesh Chand, G K. University, Hardwar, U. P.
"An extension of ancient Indian cube-root method"
50. Mr.Prem Lal Mishra, A. P. S. University, Rewa, M. P.
51. Dr. Srikant, D. R. D. O., New Delhi
52. Mr. Jai Narain Prasad, K. C. T. C. College, Raxaul-845305
53. Dr. S. K. Bhargava, 931-Civil Lines Jhansi, U. P.
54. Dr. J. S. Parihar, Atarra College, Atarra, Dist. Banda, U. P.
55. Dr. A. K. Agrawal, Sahu Jain College, Najibabad, U. P.
56. Dr. P. K. Mittal, Govt. P. G. College, Rishikesh
57. Miss Bharti c/o- Dr. Kusum Shanker, Fellow Computer Centre, Central Building, Research Institute, Roorkee-274667, U. P.
58. Miss Neeti Prakash, Meerut University, Meerut, U. P.
59. Miss Sarita Rani Sharma, G. K. Umiversity, Hardwar, U. P.
60. Mr. K. Manju Nath, Civil Engineering Department, Molrand Engineering College, Assan, Karnataka
61. Dr. Virendra Arora, Registrar, G. K. University, Hardwar
62 Dr. Rajendra Kumar, G. K. University, Hardwar. U. P.
63. Shri B. P. Shukla, G. K. University, Hardwar
64. Mr. Narendra Parashar, G K. University, Hardwar
65. Mr. Arun Kumar, G. K. University, Hardwar
66. Mr. Manoj Kumar Tyagi, G. K. University, Hardwar
67. Mr. Swami Nath Tiwwari, G. K. University, Hardwar

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MATHEMATICS IN VEDIC LITERATURE

By

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First of all, I would like to thank the organisers of this Conference for inviting me here so that I could get this opportunity of talking to this august audiance.

In recent years, there has been a lot of talk about 'Vedic Mathematics' in the country at various levels. Several activities such as seminars, conferences, work-shops, round-table conferences etc. are taking place. But there seems to be a lot of confusion and controversy as to what really is meant by Vedic mathematics. You have already heard about what prof. Puri calls Vedic Mathematics.

What I am going to talk about is the real ancient vedic mathematics. This is *the* genuine vedic mathematics as contained in the Vedic literature. Again, there is a lot of disagreement as to which period is really to be regarded as the Vedic period. But a more accepted view seems to be that it is the period 2500 B C. to 500 B C. It is my intention in this talk to give you some glimpses of mathematics as encountered in the Vedic literature of this period. In the end, I would also like to make some remarks about the Vedic mathematics about which Prof. Puri talked and the controversey surrounding the same.

Like any other branch of Science, mathematics owes a great deal to the ancient Indian literature, the earliest available works being the Vedas. There is enough evidence that in the Vedic era, there existed a high order culture and civilization. Although sciences developed in

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an attempt to help the religion, but soon they began to be studied for their own sake. Importance of mathematics was felt by all indian religions. In ancient India, the word *Ganita* used to be in vogue for the science of calculations. The following fomous verse of *Vedanga Jyotisha* indicates the special status mathematics enjoyed at that time :

"*Xatha Shikha Mayuranam, naganam manayo yatha Tadvat Vedangashastranam Ganita moordhanisthitan*"

That is, as the crests of the peacocks and the jewels of the serpents. mathematics stays at the head of all the *Vedanga shastra*. And ofcourse, who can refute the uses of mathematics. At no point of time, the civilization could do without mathematics. Not only religion, the day-to-day needs of a common man necessiated the study of mathematics in some form or the other. In *Chhandogya Upnishad*, there is a story in which Narad approaches sage Sanat Kumara and begs him to impart to him the *Brahm-Vidya*. Before doing so, the sage wanted to make sure that Narada deserved to know it. So he asked Narada to tell him what subjects he knew already. Narada then enumerated what he knew already and his list included *nakshatra vidya* (astronomy) and rasi-vidya (arithmetic). This clearly shows that *Ganita* was a subject of study during the vedic days.

Let me throw some light on what really is meant by the Vedic literature. The vedas are well-known as four in number : Rgveda, Samveda, Yajurveda and Atharvaveda. Each veda has four parts : *Samhitas, Brahmanas, Aranyaim* and *Upnishad*. As a necessary condition for mastering the Vedas, there arose six branches of study within the vedic schools : *Shiksha* (phonetics) *Kalpa* (rituals) *Vyakarana* (grammar) *Nirukta* (etymology), *Chhanda* (metric) and *Jyotisa* (astronomy). This group of literature is collectively called

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Vedangas often referred to as the appendages of the vedas. *Vedangas* were written in the form of *sutras* (a phoristic style), a form of expression characterised by great precision, brevity and economy of words, the like of which is not met within the entire literature of the world . The style was developed to sum up only the pith of the learning in short sentences using nouns often compounded at great length and avoiding the use of verbs as far as possible . The style became a dominant feature of the various branches of *Vedangas* and was also adopted by the writers of *Arthashastra*, *Kamshastra* and *Natyashastra* .

The *Kalpasutras* are concerned primarily with the rituals which constitute the chief contents of the Brahmanas. These are supposed to be the first *Vedangas* to have received systematic treatment . The *Kalpasutras* are again available in four different classes : the *Srauta-sutras*. *Grhya-sutras*, the *Dharma-sutras*, and the *Sulba-sutras* . The *Srauta-sutras* deal with the sacrifices discussed at length in the *Brahmanas* . These are naturally concerned with direction for the laying of the sacrificial fires for the fire-sacrifice, the new and the full moon, the seasonal, the Soma and other sacrifices. These are our most important source for the understanding of the indian sacrifice cult . Through their preoccupation with the laying and construction of the various altars and fires, these also constitute an important source of Vedic, and possibly the earliest mathematics in India . Of special importance as far as mathematics is concerned, are the *Sulva-sutras* which are sometimes classified as a separate branch of the *Srauta-sutras*, but which are often found attached to the *Srauta-sutras* .

The *Sulva sutras* are of special importance because these deal specifically with rules for the measurements and constructions of

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the various fires and altars and hence involve geometrical propositions and problems relating to rectilinear figures, their combinations and transformations, squaring the circle and circling the square, as well as arithmetical and algebraic solutions of problems encountered in connection with such constructions and measurements. The literal meaning of the word *Sulva* is to measure . Eventually, the word came to mean cord or rope. In ancient India, the word *Sulva* or *Rajju* (rope) was used for Geometry . The word Rekhaganita came in usage much later .

At present, we know of only seven of the *Sulva-sutras* : *Baudhayana*, *Apaat hamba*, *Katyayana*, *Manava*, *Maitrayana*, *Varaha* and *Vadhula* . These were named after the *rishis* who had written them. All the Sulva-sutras exeept the Katyayana, belong to the section of Vedas called *Krishna-yajurveda*. The *Katyayana sutra* belongs to *Shukla yajurveda*. From the mathematical view-point, Bodhayana, Katyayana and Apasthamba are of significance . These were written between 800 BC and 500 BC. Other than these seven, there is no evidence of any other Sulva-sutras being in existence .

One is simply astonished to find that the geometrical-constructions as given in the Sulvas cover the first two books and the sixth book of Euclid. More so, since the Euclid's geometry is based on certain axioms and postulates and consists of results arrived at logically using the same . On the other hand, the logical methods of Greek geometry are not to be found anywhere in Hindu geometry . Besides, the sulvas are not formal mathematical works. They appear only as parts of ancient religious works .

Baudhayana's sutras are not only the earliest but represent the most systematic, logical and detailed treatment of the subject inspite

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of their highly condensed sutra-style. *Apastamba* more or less provides the same rules and techniques. *Katyayana's* treatment is systematic. He emphasises the geometry behind the construction of altars and fires and gives a clear exposition of it. The *Manava-Sulbasutra* gives methods and details often very difficult to comprehend. In many cases, the details are either lacking or incomplete and can be understood only by reference to *Baudhayana*, *Apastamba* and *Katyayana*. On these four Sulva-sutras, the monograph by S. N. Sen and A K. Bag, published by the Indian National Science Academy in 1983 is particularly worth mentioning. They have presented the full critical sanskrit texts and the English translation of the four Sulva-sutras and modern commentaries on them. Another good reference is 'The Science of Sulba' by B. Datta published by the Calcutta University in 1932. Datta's article on vedic mathematics appeared 5 years later in 'The Cultural Heritage of India, Vol. III, Calcutta, 1937'.

There is enough evidence in the vedic literature that by 500 BC, the indians were well-versed in the use of numbers. They knew all the four fundamental operations of arithmetic i e. addition, subtraction, multiplication and division. Although these are not explicitly spelled out, but they were obviously taken for granted as common knowledge. For instance, in the *Rgveda* and the *Brahmanas* one comes across the case of dividing one thousand into three equal parts. In *Rgveda* it is not explicitly mentioned, but in *Taittiriya Samhita* it is clearly described as dividing the thousand into three. In *Satapatha Brahmana*. this operation of division performed by Indra and Vishnu is explained as follows: "For when Indra and Vishnu divided a thousand cows into three parts, there was one left, and here they caused to propagate herself in three kinds; and hence, even now if

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anyone were to divide a thousand by three, one would remain over ."

The indians in the vedic era were deeply interested in thinking and naming large numbers . They adopted 10 as the basis of numeration . The various recensions of the *Yajurveda Samhita* give names to numbers as large as 10^{12}. The *Taittiriya Samhita* gaves the following numeral denominations :

Eka (1) , *dasa* (10), *sata* (10^2) , *sahastra* (10^3) , *ayuta* (10^4) , *niyuta* (10^5) , *prayut* (10^6), *arbuda* (10^7), *nyarbuda* (10^8), *samudra* (10^9), *madhya* (10^{10}) , *anta* (10^{11}) and *parardh* (10^{12}) . The same list occurs with some alterations in *Maitrayani* and *Kathaka Samhitas* and some other places . Infact, it seems that thinking out and naming large numbers was a favourite pastime of the ancient Indian mathematicians . The same tendency is noticed in the Buddhist literature where we find a centesimal scale (i. e. *Satotara Ganana*) and the name *talakshana* for the number 10^{53} the Jainawork *anuyogadwara sutra* (100 BC) called the places in powers of 10 as *gananasthan* and mentioned large numbers upto 29 places and beyond . The jains are further credited with the conception of a time-scale called *sheersh-prahelika* (i, e. 84 hundred thousand) and the suggestion of building up fantastically large numbers in aseending powers of this figure . But let us come back to the vedic period .

The Indians in the vedic period developed a scientific vocabulary of number names using the principles of addition, subtraction and multiplicntion . The system required the naming of (i) the first group of numbers i. e. the first nine digits i. e. *eka*, *dwi*, *tri*, *catur*, *panca*, *sat*, *sapta*, *asta* and *nava*, (ii) the second group of nine nambera obtained by multiplying each of the above digits by 10, i. e. *dasa*,

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vimsati, trimsat, catvarimsat, pancasat, saptati, asiti, navati. The multiplicative principle is already indicated in forming the numbers of this group. In namig a number in which both these groups participate, additive, or additive and multiplicative principles both, are simultaneously used, for instance, *eka-dasa* (11) or sapta-vimsati (720).

The numbers were classified into two classes : yugma (even) and ayugma (odd). Also, in the Atharvaveda, there seems to be a reference to zero as also to the recognition of negative numbers.

Infact Brahmagupta, a later Hindu mathematician is credited with the use of zero in its different mathematical operations, but a text namely *pingala chhanda sutra* is said to mention a symbol for zero. Sometimes different words were used for expressing the same numeral. For instance, zero was expressed by the word *shoonya or gagan, ambar, akash*, 1 was expressed by chandra, soma, prithvi or pitamaha, likewise 2 by *ashwina*; *netra* or *chakshu* etc, 3 by *triguna, loka, trijagat* etc. 4 by *sruti, varga, yuga*, etc., 5 by *tatwa* or *erana*, 6 by *anga, ritu* or *darshan shastra* and 7 by *rishi, muni* or *swara*. Symbol-system for writing numerals was also used. In rgveda, thare is a verse which has been interpreted to mean "Give me a thousand cows on whose ears the number 8 is written." There must be some symbol for the number 'eight' which was marked on the ears of the cows-whichwas a practice followed in later times also. In another place in rgveda, a gambler says "Having staked on 'one' he lost his wife", which also indicates that there was a written symbol to denote the numer one. (Regarding measures and weights, vedic measures were, simple, meeting the needs of day-to-day life. The term *pramana* meaning measure was classified into four types, namely *mana* (measure

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of weight), *avamana* (linear measurement) and *kal pramana* (measurement of time). For linear measurements, different parts of human body were regarded as units, A man's stride was known as *prakrama*. Similarly, measure of capacity *prasrishti* meaning a handful was used. Similarly *anjali* meaning two hands joined together which was double of *prasrishti* was also in common use. Sulva-sutras refer to a variety of linear and area measurements.

The vedic literature clearly indicates acquaintance with the fundamental arithmetical operations with elementary fractions In the rgveda one finds names of a number of simple fractions such as *ardha* (1/2), *tripada* (3/4) and the *maitrayani samhita* mentions *pada* (1/4), *sapha* (1/8) *kustha* (1/12), *kala* (1/16). From the sulba-sutras onwards, we meet with terms such as *amsa*, *bhaga* to denote fractions in general. These terms are used in combination with cardinal or ordinal number names. In the case of ordinal number names, the word *amsa* or *bhaga* is often omitted. For example,

tribhaga or *tryamsa* for 1/3
pancama-bhaga or *pancama* for 1/5
dvadasa-bhaga or *dvadasa* for 1/12
pancadasa-bhaga for 1/15
tri-astama or *tryasta* for 3/8
dvi-saptama for 2/7
trayastryasta for $3\frac{3}{8}$
pancamasya caturvimsa for 1/24 of 1/5, etc.

The *Sulva-sutras* contain several examples of addition, subtraction, multiplication, division and squaring of fractions. For instance in

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Baudhayana one comes across the following passage :

"Then he measures the area of this square shaped *siti*, whose side is three *purusas* less one third. At the western side of this square is the handle whose east-west length is half a *purusa* plus ten *anguls* (i. e. one-twelfth *purusa*) and north-south breadth one *purusa* less onethird. This makes the *agni* sevenfold plus two *aratnis* and the *pradesa*'. Here two *aratnis* and the *pradesa* equal 60 *angulas* or 1/2 *purusa*. Expressed in figures, this means :

$$\text{The area of the } agni = (3 - 1/3)^2 + (1/2 + 1/12)(1 - 1/3)$$
$$= 7\tfrac{1}{2}.$$

Several such statements can be traced in the *Sulva-sutras*.

There is evidence in *samhitas* that the indians in the vedic period were interested in series or progressions of numbers also. In *taittiriya samhita*, one finds the following series :

1, 3, 5, ... 19, 29, .., 99
2, 4, 6, ... , 20
4, 8, 12, ... , 20
5, 10, 15, ... , 100
10, 20. 30, ... , 100 etc.

The arithmetic series were classified into *yugma* (even) and ayugma (odd) series. The following examples of these two categories are given in the *vajasaneyi samhita* :

4, 8, 12, .. , 48
1, 3, 5, ... , 33.

Of these two series, the second one is found to occur also in *taittiriya samhita*. In the *pancavimsa brahmana*, there is given a list of sacrificial gifts which form the following series in geometrical progression :

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24, 48, 96, 192 , ... , 49152, 98304, 196608. 393216 . This series also makes its appearance in *Srauta-sutras.* In *satapatha Brahmana* summation of such series is also given. For instance, the sum of the series .

3x (24 + 28 + 32 + ... + to seven terms) is correctly indicated as 756 . In the *Brahaddevata,* the sum of the series

2 + 3 + 4 + ... + 10000 is given correctly as 500499. Although no rule for summing such series is mentioned there, but one is led to believe that the indians in the vedic era must have known some method for summation of both, the arithmetical and the geometric series .

There are several results in the Sulva-sutras which suggest that the indians of the vedic-time knew how to perform fundamental operations with fractional numbers .

A study of vedic-literature reveals that so far as the geometry is concerned, the indians at that time bad obtained several results concerning rectilinear figures, the mensuration of areas and volumes of these figures etc . They had a special fascination for the pythagoras theorem, which in *Baudhayana* is stated as follows :

"The diagonal of a rectangle produces both (areas) which its length and breadth produce separately. " That is, the square described on the diagonal of a rectangle has an area equal to the sum of the areas of the squares described on its two sides . The converse of the pythagoras theorem, although not mentioned explicitly, was cleverly used throughout in the construction problems of fire-altars . Many a times, figures had to be first converted into squares and then the Pythagoras theorem and its converse were repeatedly used to arrive at the desired result .

Elementary treatment of surds is found at various places in the

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vedic literature. $\sqrt{2}$ has been referred to as *dvi-karani*, $\sqrt{3}$ as *tr-karant*, $\sqrt{4}$ as *catuskarani* etc. The irrationality of $\sqrt{2}$ was recognized and its approximate value as calculated then woks out to be 1.4142156 which is correct upto fifth place of decimals. It is amazing how the value of $\sqrt{2}$ was obtained to such a degree of accuracy in those early days.

In an attempt to determine the size and number of bricks needed for building the different layers of the altars. simultaneous indeterminate eqnations of the first degree were encountered. This necessiated the methods of solving quadratic equations and indeterminate equations etc. In ihe vedic-literature, one finds references of permutations and combinations. There were problems such as the total number of perfumes that can be made from sixteen different substances taken one, two, three or four substances at a time and so on.

In the closing phase of the vedic,period the Jainas deserve special notice for their interest in mathematics, Their literature makes refernce to various mathematical topics such as *sankhyayana* (Science of numbers), *parikarma* (fundamental operations), *vyavahara* (subjects of treatment), *rajju* (gaometry) *rasi* (heap, solid mensuration), *classvarna* (fractions), *yavata-tavata* (equations, algebra), *varga* (square, quadratic equations), *ghaaa* (cubc cubic equations), *varga-vnrga* (biquadtatic equations) and *vikalpa* (permutations and combinations).

Now, I would like to make a few comments on the book entitled Vedic Mathematics or sixteen simple mathematical formulae from the vedas written by Jagatguru Shri Bharti Krishna Tirathji Maharaja, Shankaracharya of Goverdhan math of Puri about which a lot has been said already. The book wrs first published by the Banaras Hindu

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University in 1965 as volume 10 of Hindu Vishwavidyalaya Nepal Rajya Sanskrit series. Subsequently, the book has been reprinted ten times, the latest edition appearing in 1988. Because of the word vedic mathematics in its title one gets the impression as if the mathematics contained in the book is the mathematics of the vedic era. However, this is not true. The results of the book are to be found no where in the vedic literature. V. S Agrawala, the general editor of Swamiji's book, was also aware of this fact. But because of his regards to Swamiji he wrote as follows :

"The question naturally arises as to whether the sutras which form the basis of this treatise exist anywhere in the vedic literature as known to us. But this criticism loses all its force if we inform ourselves of the definition of veda given by Swamiji himself as quoted below :

'The very word veda' has this derivational meaning i. e. the fountain-head and illimitable store-house of all knowledge. This derivation, in effect, means, connotes and implies that the vedas *should* contain within themselves all the knowledge needed by mankind relating not only to the so called 'spiritual' or other worldly matters but also to those, usually described as purely purely 'secular', 'temporal' or 'worldly' and also to the means required by humanity as such for the achievement of all-round, complete and perfect success in all conceivable directions, and that there can be no adjectival or restrictive epithet calculated (or tending) to limit down that knowledge in any sphere, any direction or any respect whatsoever. In other words, it connotes and implies that our ancient Indian Vedic lore should be all-round, complete and perfect ard able to throw the fullest necessary light on all matters which any aspiring seeker after

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knowledge can possibly seek to be enlightened on" . It is the whole essence of his assessment of vedic tradition that it is not to be approached from a factual stand point, but from an indeal stand point i. e. as the vedas as traditionally accepted in India as the repository of all knowledge *should be* and not what they *are* in human possession. That approach entirely turns the tables on all critics, for the authorship of vedic mathematics then need not be laboriously searched in the texts as preserved from antiquity" .

In his preface to his book, Swamiji has stated that the sixteen Sutras dealt with by him in that book were contained in the *Parishishta* of the Atharvaveda. But this is also not a fact, for they are not to be found anywhere in the known *parishishtas* of the *Atharvaveda* edited by G. M. Bolling and J Von Negelein . According to Professor K. S. Shukla, sometime in 1950's when Swamji visited Lucknow to give a black-board demonstration of the sixteen Sutras of his book at the Lucknow University. Prof. Shukla personally wenr to him at his place of stay with Bolling and Negelein's edition of the *Parishishtas* of the *Atharvaveda* and requested him to pinpoint the places where the sixteen Sutras demonstrated by him occured in those *parishishtas* Professor Shukla narrates that Swamiji replied offhand, without even touching the book that the sixteen Sutras demonsitrated by him were not in those *parishishtas,* they occured in his own *parishishta* and not in any other .

V. S. Agrawala has also given the verdict that the work of Swamiji deserves to be regarded as a new Parishishta by itself . However, the question is whether any book written in modern times on a meodern subject can be regarded as a Parishishta of a Veda. The answer is definitely in the negative. It may be '*good* mathematics' but certainly not '*vedic* mathematics' .

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According to this view, it seems that the sixteen Sutras of Swamiji's 'Vedic mathematics' are Swamiji's own creation and have nothing to do with the mathematics of the vedic period. Infact, Swamiji goes a little too far in his claim. He designates his preface to the book as 'A descriptive prefactory note on the astounding wonders of Ancient Indian Mathematics'. Not only that, at several places he calls his mathematical processes as vedic processes. Now the point is that a general reader has no access to the vedic literature. It is therefore not possible for him to actually verify Swamiji's claim that the *Sutras* as given in the book appear somewhere in the vedas. No wonder then that there is a general impression (unfortunately even at a high level) that the mathematics as contained in the vedas consists of the sixteen *sutras* of Swamiji's book.

Professor K.S. Shukla comes out in a big way refuting Swamiji's claim. He has quoted several instances from the book supportiug his views, some of which are as follows :

In chapter one, conversion of vulgar fractions into recurring decimal fraction is given Nobody in the vedic period could think of a recurring decimal. The methods of multiplication and division are quite different from the traditional Hindn methods. The theorems of Leibnitz, Maclaurin aad Taylor, integration by partial fractions dealt with in the book are all modern topics. The system of expressing numbers by means of letters of the Sanskrit alphabet has been called by Swamiji 'the vedic numerical code' although it has not been used anywhere in the vedic literature.

Dr. Puri remarked that how can we say that these Sutras are not contained in the several lost unpublished *pandulipies*. But as I understand, vedas are our sacred books which have not been allowed

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to be tempered with. Either the Sutras are there and if they are not there, they cannot be called vedic. Mathematicians do not make statements in the air. Nor do they believe such statements. For everything, there must be supporting evidence-either a proof or a counter-example. As Professor Kapur also said and as you must also have felt in view of what I have said that here is a vast gap in the type of mathematics contained in vedas and the vedic mathematics of ancient time. Also there is no circumstantial evidence available in support of Swamiji's claim. One should be able to show that if not explicitly, atleast implicitly the Sutras are there in the vedas. Unless that has been done, I don't think there is enough justification for calling it vedic mathematics.

Inspite of all this, one must accept that the *S*utras given in the Swamiji's book are beautiful, They do make calculations amazingly simple and fast in some particular. The book has inspired work in some new directions The late Professor P. L. Bhatnagar had done an indepth study of Swamiji's book and published a long paper in the Mathematics Teacher. In this paper he tried to *prove* some of the *Sutras*. Some more work has to be done in that direction, There is a suggestion of introducing *Sutras* as part of regular school curriculum. But here again we have to be careful. Unless we can show the universal applicability of the results, they cannot be introduced as part of the syllabus. As enrichment material. one con recommend Swamiji's book without any hesitation. Efforts are afoot to establish and investigate the origin of the sixteen Surras.. At a recent round table conference in Delhi organised by the *Bashtriya Veda Vidya Pratishthan*, there was a suggestion to study these sutras as vedic mathematics of J. S. The *Pratishthan* had earlier organised two national workshops-one at Jaipur and the other at Ahmedabad. As

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regards the literature, the National Commission for the compilation of the History of Sciences in India is doing commendable work. Due to its efforts, some good books have been published. For example, 'A concise history of Science in India' edited by D.M Bose, S.N Sen and B.V Subharayappa published by the Indian National Science Academy in 1971 is an excellent sourcematerial. It gives a concise account of the development of Science in India from ancient times to the 18th century. There are 11 chaptars on different subjects in the book, mathematics being one of these. Another book in the series on sulba-sutras by Sen and Bag published by INSA in 1983 I metioned earlier.

For the last 30 years, A. Seidenberg has been making serious study of vedic mathematics at part of his general theory of ritual origin of mathematics. His 42 page paper on 'The origin of Mathematics' (Arch. Hist. Fxact Sciences, 1978) includes a speaial Appendix on 'Vedic Mathematics',

To sum up, the *genuine* vedic mathematics is that mathematics which is contained in the vedic literature in the period 2500 BC to 500 BC. The indians in the vedic period werc well-versed in the use of numbers. Large numbers sueh as 10^{12} were known to them They had developed a scientific vacabulary for number names. Symbols were also used for writing numerals, They knew all the four fundamental operations of Arithmetic. They were familiar with simple fractions. They were interested in series of numbers. In geometry, they solved propositions about the construction of various rectilinear figures, mensuration of areas and volumes etc. No doubt, this vedic mathematics is of great historical significance for studying and understanding the science and culture of India about 2000 to 5000 years ago. The modern system of vedic mathematics as

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propounded by Jagadguru Shankaracharya, howsoever beautiful it might seem to be, should not be confused with the real ancient vedic mathematics a glimpse of which I have tried to give, One must also be cautioned against the exaggerated extreme belief that all modern discoveries and inventions are hidden secretely in the vedas. One must more with care, Let us understand clearly : Vedic mathematics is only that mathematics which is contained in our sacred vedic literature .

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VEDIC MATHEMATICS - MATHEMATICS OF SMILES

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INTRODUCTION

I wish to draw your kind attention to another wonderful mathematical facet of rich, vast and scientfic cultural heritage of India. Vedic Mathematics, derived from the Veda, provides one line, mental and superfast methods alongwith quick cross checking system. As such Vedic Maths is a boon for all competitions. Vedic Maths converts tedious Maths into a playful and bliss ful subjcct, which students learn with smiles.

SALIENT FEATURES

Vedic Maths (VM offers a new and entirely different approach to mathematics based on pattern recognition for resolving the current crisis in Maths education allover the world. VM allows for constant expression of a student's creativity and has been found to be easier to learn For any problem there is always one general technique applicable to all cases, There are also a number of special techniques for special pattern problems The element of choice and flexibility at each stage keeps the mind lively and alert and develops clarity of mind and intuition, and thereby Holistic development of the hnman brain automatically takes place.

UNIQUE FEATURES-IN BRIEF

1. BINDU MEIN SINDHU

Vedic Approach of Bindu Mein Sindhu : In Vedic thought we

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always think of large number of possiblities and we consider that each small drop essentially represents an ocean-Bindu Mein Sindhu. Further, both negative and positive coexist to make the play go on. Perfectly, in tune with this, we find that in Vedic Mathematics, we have both positive and negative digits and a number consisting of both positive and negative digits is called vinculum number .

As we learn the numbers we also learn their complements . This then immediately provides a possibility of seeing any digit in two alternative forms. For exomple

$$8 = 10 - 2 = 1\bar{2} .$$

The question naturally comes up if there is some other possibility to express 8 which still the value of 8 .

VM provides yet another possibility as

$$1 = 1\bar{2} = 1\ \bar{9}\ \bar{2}\ (100 - 92)$$

LINK WITH ALGEBRA : Once we become very familiar with the vinculum numbers and their operation, then we can see a perfectly convenient link between Arithmetic and Algebra . A polynomial

$1x^3 - 9x - 2$ is just one algebraic representation of number 8 . (with $x = 10$, as we are working in the decimal system) . as $1x^2 - 9x - 2 = 1\,\bar{9}\,\bar{2}$

Therefore, in Vedic system, all operationai procedures of Arithmetic are straight away used in Algebra which provides tremendous amount of simplicity, ease and convenience to the student . Further, Vedic Mathematics provides the clear illustration of the Vedic concept, 'Bindu Mein Sindhu' .

2. MATHEMATICS - A VITAL SUBJECT

Whatever we want to do in life we always have a large number of

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alternatives and choices available to us. This element of choice makes the mind lively and alert , However if we are asked to do the same thing in a fixed order following a rigid sequence of steps, even a very convenient and interesting task shall soon become boring and tedious and is very likely to result in unnecessary anxiety and boredom. On the other hand, the availability of several alternatives and choices gives the opportunity of freedom of selection and makes the whole task interesting. This element of choice makes the mind alert and agile, as the mind tries to follow the path of least resistance and gives the individual opportunity of self expression in varied ways .

For simple arithmetical operations we have a unique fixed and general method of addition, subtraction, multiplication, and division. Same is true for most of the higher aspects of Mathematics. We spend well over 5 years mastering these four basic arithmetical operations. This fixity and rigidity with little possibility of any type of annovation, freedom and choice leads to monotony which in turn creates the unnecessary problem of MATHS ANXIETY. A common student is afraid of Maths . He is afraid of the most important, fundamental and vital subject - MATHS.

Children love to play games . As a matter of fact we all like one or the other game . If we could teach the child one method of multiplication in class 2nd. Give him a second method of multiplication in class 3rd; add another alternative after six months for very conveniently multiplying the numbers closer to the base; teach him multiplication using Vinculum numbers (consisting of both positive and negative digits) in class 4th; which would require least computational effort; and after he has practised well, multiplying from right to left, the student learns the procedure of multiplying from left to right . With the result that even process of multiplication

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becomes a game for a poung child. This exactly has been the response of the students of the schools of England, Holland and America, where Vedic Mathematics (VM) is being taught .

Dr. Robin Rowe Academic Director of group of MSAE schools at Fairfield in Iowa state (U. S. A.) reports; "The children are bubbling with *joy and bliss* in Vedic Maths class" .

So we find that Vedic Mathematics with its special features has the inbuilt potential to solve the problem of Maths Anxiety .

3. MOST EFFICIENT COMPUTATIONS

All of us have this experience, that whenever we are operating the bigger digits (6, 7, 8. 9), it requires slightly more effort and a little more time, compared with the operation of smaller digits . This difference may be only in microseconds , but definitely it is slightly more straining to operate bigger digits ,

The recent medical research on human brain has lead to the conclusion that the information in human brain is stored in memory registers . The front most register can store only five bits of information. Therefore, when we are using the first five digits (1 to 5) only the front most register is used, hence the ease and convenience, but for operating the bigger digts (6 to 9) the second level register of human memory has to be used. As such it requires a slight more effort and time .

The whole Vedic tradition is based minimization of efforts on the part of the human being, which is the roof and crown of God's creation . Further. in VM it is possible to do the entire computations by using only the front most register waich can store five bits of information .

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In Vedic Maths we have a choice to use the digit 8 as it is or alternatively use $\bar{1}2$. Similar is the case with other digits. These are referred to as Vinculum numbers.

Therefore, in VM it is possible to carry out the entire computation process using only the front most memory register by operating the smaller digits. We shall all agree that operating 1 and $\bar{2}$ is far more easier, simpler and quicker than handling 8 Further, the chances of error are drastically reduced. As a matter of fact when we are operating only smaller digits we also have an opportunity to smile.

4. MULTIPLE CHOICE

In the present system of Mathematics we have most of the time, a unique way of solving the problem. This has the obvious advantage of convenience of learning the procedure in a shorter time, but this unique fixed way of working very often leads to monotony. There seems to be no possibility of improvement of computational efficiency in any way. Vedic Maths on the other hand provides multiple ways of solving any problem. Once we learn the three Vedic ways of multiplying numbers. The mind naturally thinks there should be some other way also and soon we find that we can multiply in six different ways. We have a choice to work from left to right or right to left. Further, we can convert the bigger digits into Vinculum form.

5. CHECKING METHOD

One can easily check the correctness of the result obtained by redoing the problem using another way of solving.

Alongwith the multiple choice of working, VM also provides us with independent cross checking methods. These checking methods provide a convenient opportunity to smile at the end of each problem.

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6. COMBINED OPERATIONS OF ARITHMETIC

VM emphasizes the mental working and provides an opportunity to carry out the combined operations of Arithmetic in a single step. With practice the extent of combinations keep on growing. Sum and difference of the products of a few pairs of numbers can be done directly in one line etc.

The possibility to carry out the combined operations of arithmetic has tremendous applications in different areas of Modern Algebra and Higher Mathematics .

7. OBTAINING DIRECT SOLUTION

Presently, in many operations like multiplication, squaring, cubing, adding etc., we work form right ro left. The left most digit is almost always the most significant digit and the second digit from left is the next significant digit . Since we have to work from right to left, it is almost necessary to carry out the entire computations for getting the most significant part of the answer . In Vedic system we have a complete choice to start our computations either from left or from right (AVM Pushp 2 and 3) . It just does not matter whether we are multiplying or squaring or solving a transcendental equation. The capability to start our working from the left can be very usefully utilized in handling problems of diversified areas of Mathematics . Once we start our computations from the left, we are in a position to obtain the most significant digit in the very first instant . Then we use just the digits already obtained for computing the second most significant digit in the very first instant . Then we use just the digits already obtained for computing the second most significant , In this way we can go on with our working till we obtain the desired level of accuracy and precision .

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8. HIGHER APPLICATIONS

Vedic Mathematics system provides an integrated approach to different areas of Mathematics As the same sutras are applied directly for diversified areas like Trigonometry-Plain and Spherical, Positional Astronomy, Coordinate Geometry 2D and 3D, Differential and Integral Calculus etc . We only need to add very few additional concepts and formulae . The recent studies indicate that most of the problems in these areas can be directly solved and it appears that bulk of the formulae and identities which we use at present become redundent (Refer Higher Applications of VM) .

9. INDEPENDENT CHECK ON COMPUTER RESULTS :

Vedic Mathematics is the only convenient and useable system which can provide an opportunity of independently checking the computer results . Further, in several special pattern problems the results are obtained with a dramatic speed due to the simple and convenient Vedic procedures . Therefore, Vedic Mathematics system really appears to be the system compatible with the present day computer age .

10. DEVELOPMENT OF INTEGRATED PERSONALITY

According to the brain specialists, the left half of the brain. systematically collects the information, does sequential analysis and arrives at the logical conclusions and results . The right half of the human brain works with pattern recognition and has the capability of intuition . This most powerful facet of human personality the intuitive faculty, unfortunately, remains undeveloped in most of us. Can right brain thinking be taught ? Yes, say the experts but it takes

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time and practice. With the left brain silenced the holistic right moves into action .

The Vedic Mathematics system provides us with a large number of options at each stage of working. Depending upon the pattern of the problem, one of these options is going to give the result with very little effort and in minimum time . Therefore, the very first step is to look for the pattern of the problem, which essentially involves systematic use of the right half of the brain . Then the logical computations are done by the left half . Even while computing, due to various options available, the mind is always kept alert to pick up the path of least effort . This in turn brings joy in the heart and smile on the face of the students . Therefore while practising Vedic method human brain gets the systematic training of integrated functioning of the two halves of the brain . This according to the educational experts is very likely to result in Holistic Development of Human Personality. In this context, the feed back from the American schools, where systematic teachings of Vedic Mathematics has already been introduced is thought provoking .

The head of Maths department from St. James School London (where they are systematically and *exclusively teaching Vedic Mathematics since last 8 years*) has informed that, ' The general performance of our students in Maths is excellent and it is helping them in development of their personality. Because the Vedic Methods take proper account of how the hnman mind works the calculation is done faster and leaves the mind bright and full of energy . All of us find that the effect on the boys is to make them very bright .

One boy shouted back an answer so quickly that the teacher asked, "How did you do it ?" He said "It just flashed in my awareness" This is the working of the intuition faculty .

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In a nutshell we can say that the Vedic Maths provides the *cosmic software* for the *cosmic computer* (*the human brain*) creating coherence in the brain physiology, so that the later is not wasted ,

CURRENT DEVELOPMENTS IN INDIA

The interest in Vedic Mathematics, as reexpounded by Shankra Charya Bharti Krsna Tirth ji, has been steadily growing in the last few years . The book by Shankracharya was originally published in English by the Banaras Hindu University . Its translations in Marathi and Gujarati have already been published . Further, 20 books have been published in India and 5 books in U. K. ,

(i) Ministry of Human Resource Development (MHRD) has started the Rashtriya Ved Vidya Pratishthan (RVVP) which has already organised two national workshops, and a Round Table discussion on Vedic Mathematics . RVVP has set up a national committee to plan further Vedic Maths activities .

(ii) The National Council of Educational Research and Training (NCERT) has identified Vedic Mathematics as one of the thrust areas and included Vedic Mathematics ia the teachers training course and teachers manuals for enrichment material in Maths education .

(iii) The Vedic Maths Research Group (VMRG) was formed at the Roorkee University in 1988 and is organising courses and research in Vedic Mathematics . The Spiritual Study Group (SSG) of Roorkee has been active in organising VM lectures, short courses and correspondence courses in different parts of the world . SSG has published series of books and released video and audio cassettes .

(iv) A series of training courses and workshops are being organised in Madhya Pradesh. Maharashtra, Gujrat, Rajasthan and

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Uttarpradesh. Series of Vedic Maths programmes are being shown under UGC National class room, T. V. programmes .

(v) Systematic teaching of Vedic Mathematics has been started at a few schools . The secondary education department of Madhya pradesh has permitted teaching of Vedic Mathematics to class IX and X at the Government schools at Mawai, Tikamgarh on experimental basis .

VEDIC MATHEMATICS RESEARCH GROUP
ROORKEE UNIVERSITY

The Vedic Maths Research Group (VMRG) at Roorkee university was formely established in october 1988 with Prof . Anand Swarup Arya, Pro-Vice Chancellor, Roorkee University as the Chairman . Prof . S. N. Pandey, Head of the Maths department and Prof . G. C. Nayak, Head of the Civil Engineering Department and coordinator CAD-CAM centre of Roorkee University as the Co-chairman. Dr. N. Puri. Reader in Civil Engineering Department is the Secretary . Further, VMRG has members from the departments from Civil Engineering, Computer and Electronics Engineering, Mathematics, Mechanical Engineering, Metallurgical Engineering, Hydrology and Architecture .

The Spiritual Study Group (SSG), consisting of the students and staff of the Roorkee University and now having several outside members, started the study of Vedic Mathematics in March 1985 . The response of the participants was excellent . This resulted in a series of Vedic Maths courses .

SSG has further started the Instiute of Vedic sciences (IVS) . IVS has been conducting short courses and correspondence courses on

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Vedic Mathematics. SSG has been publishing a large number of books and workshop notes on Vedic Mathematics. Further, a series of Video and Audio cassettes on lectures and courses of Vedic Mathematics have also been released .

The VMRG has been conducting the Vedic Maths courses for students and teachers . The members of VMRG prepared a set of 8 research papers on Higher Applications of Vedic Mathematics for the Ahmedabad workshop of RVVP. VMRG has prepared and submitted major research projects on, Vedic Mathematics and its applications in Civil Engineering and Computer Graphics, for the University Grants commission and Department of Science and Technology of the Government of India .

VMRG is also preparing special books for Engineers, teachers, bankers etc.

The MHRD established the Rashtriya Ved Vidya Pratisthan (RVVP) in 1987. MHRD, RVVP, University of Rajasthan alongwith the Pondichery University organised the first national workshop on Vedic Mathematics at Jaipur in March 88 . The Workshop accepted Vedic Mathematics as a very interesting and useful subject and recommended the setting up of a National committee . It felt the need to provide funds for further research and development of the subject and spread the available information .

This was followed by the second workshop held at Ahmedabad in Oct. 88. 8 Theme papers on applications of Vedic Mathematics in Algebra, Trigonometry, Coordinate geometry, Calculus and solution of linear and non linear integro-differential equations were prepared by the Vedic Maths research group of Roorkee University for the workshop, The question of introducing Vedic Mathematics at the

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primary level was also discussed alongwith higher applications of VM. The simplicity and case provided by Vedic Maths in trigonometry, both plane and spherical, was highly appreciated. The MHRD further held the Round Table discussions (RTD) on Vedic Mathematics in Feb . 89 at Delhi . The RTD gave the following recommendations .

The Round Table Discussion organised by the Rashtriya Ved Vidya Pratishthan from 22nd February to 24th February, 1989, has come to the conclusion that the Sutras in the book 'VEDIC MATHEMATICS' of Jagat Guru Shankaracharya Shri Bharti Krsna Tirtha ji, are interesting and useful . While further theoretical and experimental investigations are necessary before a decision can be taken for introduction in the curriculum, the following positive steps are recommended :

(i) The national council of educational research and training may bo requested to include some of the sutras and their adplications as enrichment materials, in consultation with a panel of mathematicians constituted for the purpose by the Rashtriya Ved Vidya Pratishthan.

(ii) Agencies inclined to conduct training programmes in Vedic Mathematics should be encouraged and supported .

(iii) The Rashtriya Ved Vidya Pratishthan should commission the writing of the following-

(a) A book giving for each sutra, its statement, its proof, its motivation, illustrative examples, explaining limitations and possibilities for generalisation .

(b) A book explaining how students can discover the sutras by recognising patterns with the help of illustrations .

(c) A book explaining sutras for educating teachers and other

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users like managers, bankers, engineers, etc .

(d) A book comparing the Sutras with other methods of calculations and in particular, with those of trachenburg speed system of Basic Mathematics .

The above books should be Wetted by a panel of mathematicians before permitting their publication .

(iv) Projects may be sanctioned to competent mathematicians and computer scientists .

(a) Development of ideas in the book 'Vedic Mathematics' by Jagat Guru Shankaracharya .

(b) Investigating the possibilities of incorporating the relevant ideas in appropriate computer software programmes .

(c) Expounding and developing Mathematical Foundations behind the sutras ,

CURRENT DEVELOPMENTS ABROAD

Vedic Maths work was started in England in 1980. The Vedic Maths research Group at London has been organising lectures/courses and published a set of 5 books . The St. James school London has been systematically and exclusively teaching Vedic Mathematics from Class-I to VII for well over 5 years . Two more schools ; one in Dublin and the other in Amsterdam have also started the teaching of Vedic Maths .

Dr. N. Puri went on a comprehensive 18 nation tour of Vedic Mathematics in early 1988 . This was followed by two regional VM training courses in June and July, 1988 at Switzerland and America . As a follow up of this a number of Vedic Maths lectures/courses are

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being organised in different parts of the world. Further, systematic teaching of Vedic Maths have been started in a number of schools at IOWA, Washington D. C., Mexico etc. Excellent response is being received from all places.

VEDIC MATHEMATCIS RESEARCH GROUP AT U. K.

The VMRG at UK has been active since 1980.

Courses held

University of London Union, 1980-1983.

Polytechnic of North London : Easter 1982 : 1983 / 84 and 1984 / 85.
Mary Ward Centre : 1985 / 86 and 1986 / 87.

Working Men's College : 1985-86.

First Vedic Mathematics visit to India (1981 by A. P. Nicholas.
Tour of India : Visiting Universities etc. seeking information, giving lectures

From : 22nd July 1981 To : 23rd Sept. 1981.

Books : Five books have been brought out from U. K.

Tour of U. S. A.

From : 7th Sept. 1983 To : 21st December 1983

Lecture / Workshops were given at the 18 institutions / universities

Vedic Mathematics Lecture Tours of Holland (17th October - 4th November, 1988) and West Germany (7-11th November and 6-24th March 1989).

Research Paper :

Pickles, J. H. Nicholas, A. P. and Williams, K. R., (1982) "Bharati Krsna's Systems of Vedic Mathematics", The Institute of Mathematics and its Applications, Vol. 18, 233-236.

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INTERNATIONAL CORRESPONDENCE COURSE ON VEDIC MATHEMATICS

The correspondence course on Vedic Mathematics is being run by the SSG, Roorkee since Haridwar Mahakumbh 1986 in English and Hindi. The course was of six months duration and consisted of two parts. From 1990, the course shall be of 1 year and shall have 3 parts . There are 6 fortnightly lessons in each part. New batches start from auspicious Baishakhi (April) and Dashahara (October)every year.

The first part of the course covers basic arithmatic . The second part covers basic Maths and Algebra. The third part covers the Vedic methods for trigonometry (Plane and Spherical), Coordinatc Geometry (2D and 3D), Determinants, Calculus and solution of linear, nonlinear differential and integro-differential equations, It also includes the scheme for systematically teaching Maths in the vedic way ,

A number of participants from different countries have also been joining the correspondence course .

VEDIC MATHEMATICS WORLD ACROSS

Dr, Narinder Puri went on a 7-week world tour in March and April, 1988. He visited 18 countries of Europe and Ameriea including England (London, Oxford, Cambridge, Lancashire), USA (Washington, Boston-Harvard Univ., Chicago, Los Angeles, Stanford University, PolaAlto, Berkeley, San Francisco, Indiana, Fairfield, MIU), Canada (Toronto University) Mexico, Norway (Oslo and Lily Hamer, Sweden (Stockholm and Upsala University), Finland (Turku University), Denmark (Copenhagen University and H. C. Oersted Institute), Germany (Hemburg University, Berlin . Free Univ and Technical Univ.), Hungary (Budapest, Royal Institute of Leadership). Austria (Vienna University), Italy (Rome, Naples University, Milan

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University and Como), Switzerland (Seelisberg MERU, Geneva University), France (Institute Henri Poincane-Paris), Spain (Barcelona University), Holland (MERU) and Belzium . Dr. Puri delivered 101 hours lectures during this tour. Dr. Puri was interviewed by several TV, Radio and Press media during his tour. After magnificant developments in Hardware and Software technology, there is an urge for Brain ware development. It requires a totality of brain, where the Vedic Mathematics plays an important role. Everywhere the Vedic Mathematics was received with much reverence and hopes. The Maths educators world over felt that the Vedic Maths is the wonderful system to develop the total brain of students and solve the basic problem of Maths anxiety being faced in Maths education the world over.

CONTINENTAL COURSES

International courses of two weeks each, on Vedic Maths were organised during June and July, 1988 at MERU Seelisberg, Switzerland, Mexico and M. I. U. IOWA, U. S. A. All the lectures were delivered by Dr N. Puri, Reader in Civil Engineering, University of Roorkee. The MERU course was joined by participants from England. France, Spain, Holland, Germany and India. The U. S. A. and Mexico courses were joined by participants from Mexico, Panama, Canada and different states of America. 88 hrs of Video recording was done. These are Planned to be offered as future courses in different parts of the world. All the participants were highly impressed by the unique features of Vedic Maths. Several schools in USA etc. have started the teaching of Vedic Mathematics at different levels .

The initial feed back from different parts of the world is excellent . The participants of these courses have started delivering lectures and conducting courses in different parts of the world inclu-

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ding China, Hongkong, Canada, USA, Japan, Sweden, France, Lebanon, England and Holland .

GLIMPSES :

1. NIKHILAM MULTIPLICATION

We have learnt the second Vedic sutra (Nikhilam Navatascaram Dastah) means all from nine last from Ten . Nikhilam method of multiplication can be used for multiplying numbers with big digits .

Example 1 : Multiply 9 × 6.

Step 1 : Select nearest base . Here base ia 10 . Base is the number with left digit as 1 and all other digits as 0, Therefore 10, 100, 1000, 10000 etc. are different bases .

$$\begin{array}{c} 9 - 1 \\ 6 - 4 \\ \hline 5 / 4 \\ \hline \end{array}$$

Step 2 : Compute the deviation of each number from the base , Write the deviation with sign in front of the number as shown. We use Nikhilam sutra . Deviation of 9 is 1 (last from ten 10-9). Since 9 is smaller than the base 10, we write this with the negative sign. Similarly, deviation of 6 is 4 (10-6=4) , which is the same as the purak of the last digit from 10 . Since 6 is smaller than the base, we write the deviation with a negative sign ,

Step 3 : Divide the result space by drawing a slash .

The right hand side of the result is simply the multiplication of the two deviations . Therefore $\bar{1} \times \bar{4} = 4$.

This is written on the right hand side side of the slash .

Step 4 : The left hand side of the result is simply the addition of any number with the deviation of the second number .

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(i) Therefore $6 + \bar{1} = 5$ 　　　9 - 1

or 　　　6 - 4

(ii) $9 + \bar{4} = 5$ 　　　5 / 4 = 54

The resulting 5 is written on the left hand side ,

Hence 54 is the answer .

Example 2 : Multiply 8×9 .

Step 1 : Base is 10. 　　　8 - 2

Step 2 : Compute the deviations . 　9 - 1

7 / 2 = 72

The puraks are $\bar{2}$ and $\bar{1}$.

Step 3 : Multlply the deviations $\bar{2}$ and $\bar{1}$ so $\bar{2} \times \bar{1} = 2$ (R. H. S.)

Step 4 : Cross and $8 + \bar{1} = 7$ or $9 + \bar{2} = 7$ (L. H. S.).

Hence 72 is the result .

Even subbases can be used (Refer AVM Pushp 2)

9 7 - 3	1 0 4 + 4
9 6 - 4	1 0 3 + 3
9 3 / 12	1 0 7 / 12

2. MULTIPLICATION - THREE OR MORE NUMBERS

The superfast Vedic procedure of multiplication can be further extended to multiply three, four or even more numbers simultaneously. Of course, the procedure becomes just a bit longer for larger nnmbers. This is another very special and unique feature in the cap of the 'Nikhilam Navatascaramam Das atah' Vedic Sutra The following few examples shall serve the purpose of

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clearly demonstrating the very simple, straight forward step by step procedure .

As we already know. 'Nikhilam sutra'means all from 9 and last from 10.

Type – 1

Let us take a simple example of multiplying 98 × 97 × 96.

Step 1 : We write these numbers in the Vedic base system style. We get the deficiency of each number from the base by invoking the 'Nikhilam sutra' . Thereforc. we get

(The base is 100)

98 - 02
67 - 03
96 - 04

/ /

Step 2 : For three numbere the product shall have three parts, Therefore two inclined lines (//) are drawn to demarcate the three parts of the answer .

Step 3 : We vertically multiply the three deficit mumbers and the resulting number is the right hand portion of the answer In this case $(-02) \times (-03) \times (-04) = -24 = \overline{24}$ is the required right hand portion. This is written down in the right hand space .

98 - 02
97 - 03
96 - 04

/ / $\overline{24}$

Step 4 : The middle portion of the answer is obtained as follows . We multiply two deficit numbers at a time by rotation. The summation

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of these three small products gives the middle portion of our answer and is written within the two slashes ,

In the present example we get,

$(-02 \times -03) + (-03 \times -04) + (-04 \times -02) = 6 + 12 + 8 = 26$

```
 98-02
 97-03
 96-04
 -----
/26/2̄4
```

Step 5 : The left hand portion of the resulting product is obtained by the algebraic addition of the first number with the deficiency of the other two numbers .

In this example the L. H. S. is $98 + (-03) + (-04) = 91$,

The resulting number is written at the appropriate space .

```
 98 - 02
 97 - 03
 96 - 04
 -------
91/26/2̄4
 -------
```

NOTE : The left hand portion of the product can also be obtained in several alternative ways. This, to mention again is the unique facet of Vedic mathematics and each approach provides an easy check .

3. INTRODUCTION

The previously illustrated examples in chapter 6, were all solved by using the Nikhilam sutra ,

However, there are a few more corollaries arising out of the Nikhilam Sutra discussed in the previous chapter . Some of these have been presented in chapter 9 of Pushp 1 .

In mathematical astronomy we frequently require the multiplication of multipication of multiplier digits consisting entirely of nines

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with other numbers (multiplicand). For multiplication of this type the subsutra Ekanyunena-Purvena is used,

Case I : Multiply 6×9 .

According to the sutra the multiplicand has to be decreased by 1 (this gives the L. H S.) and for the R. H. S. subtract the L. H. S. from the multiplier (get complements of nines)

Therefore 6×9 = (6 - 1) / 9 - (6 - 1) = 5/4 = 54

Illustrations

(i) 9×9 = (9-1) / 9 (9-1) = 8/1 = 81

(ii) 12 × 9 = (12-1) / 9-11 = 11/$\bar{2}$ = 10/10-2 = 10/8 = 108

(iii) 13×99 = (13-1) / (99-12) = 12/87 = 1287

(iv) 19 × 99 = 18 / (99-18) = 18/81 = 1881

(v) 778 × 999 = 777/222 = 777222

(vi) 9853×999 = 9862 / 0137 = 98620137

COMMENT While obtaining the R. H. S. of the answer by subtracting the L H.S. from the multiplier (9999) all the digits including zero must be written (9999 - 9862) = 0137 ,

(vii) 3728387 × *9999999* = 3728386 / 6271613 = 37283866271613

(viii) 12345678912 × *999999999* = 12345678911 / 87654321088 OR
= 1234567891187654321088

For details refer AVM Pushp 2.

4. DIVISION - NIKHILAM METHOD

INTRODUCTION

We have seen how NIKHILAM Method can be used for the multiplication of big numbers. Some other applications of the Nikhilam

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Sutra in Vinculum numbers, Table etc. have also been discussed. It is interesting to learn that the Nikhilam sutra can also be used for the division. Like multiplication the Nikhilam method is most convenient when the divisor consists of the big digits. Bigger the digits easier is the solution by Nikhilam method.

Example : Divide 216 by 89.

89_/ 216

Step 1 : Using Nikhilam we modify the divisor.

Here MD = 1 1

Step 2 : We write as 89_/ 216
11

Step 3 : We draw a slash after 2 digits and number of the digits of the dividend from R. H. S. and the digits of divisor are numbered from L. H. S.

```
      3rd 2nd  Ist
8 9_/  2 / 1   6
1 1
Ist 2nd
```

Step 4 : Multiply the last digit of D (here 2) with the first and second digits of MD respectively and place the product below the previous digits (2nd and Ist column respectively) as follows

```
8 9_/ 2/1 6
1 1     2 2
      -----
      2/
```

Step 5 : On adding the digits of 2nd and Ist column we get the result.

```
8 9_/ 2/16
1 1     22
      ----
      2/3 8
```

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So Quotient = 2
Remainder = 38

5. YAVADUNAM METHOD

There is yet another method of squaring. This upsutra reads as follows Yavadunam Tavadunikrtya Vargarca Yojayet which means whatever the extent of its deficiency lessen it to the same extent and set up the square of the deficiency (Refer Pushp 1 Chapter 9).

ILLUSTRATION 1. Calculate the square of the number 98.

Step 1 : First we select the base nearest to the number. Here 100 is the base.

Step 2 : We divide the result in two parts by drawing a slash

$$98^2 = L.H.S/R.H.S.$$

Step 3 : The deficiency of the number from the base is (100 - 98 = 02) = 02,

Step 4 : Now we subtract this deficiency from the number to get the *L.H.S.* as 98 - 02=96.

$$\text{So } 98^2 = 96 / \text{R. H. S.}$$

Step 5 : For R. H. S. result we take the square of the deficiency. So the square of 02 is 04. Hence the result is $98^2 = 96/04 = 9604$. As there are 2 zeros in the base (100) we must have 2 digits on R. H. S.

ILLUSTRATION 2 : Calculate 996^2.

L. H. S. 996 - 004 = 992
R. H. S. $(004)^2 = 016$
Hence $996^2 = 992/016 = 992016$

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7. BEEJGANIT

In Vedic Mathematics Beejganit is the original name for algebra. Beej essentially means the seed and Ganit means Mathematics. Therefore, Beejganit should essentially be the seed of different facets of Mathematics, This is quite clear and obvious if one studies mathematics the Ancient Vedic way. Now we shall introduce ourselves to some interesting features.

Like decimal system (the name itself derived from 'Dasham' - meaning ten in Sanskrit) the algebra has also originated from India and in a recent research study swami S. A. Paramhansa has explained how even the name algebra was developed from original Beejganit (presented in tbe Jaipur workshop).

Let us consider a polynomial $2x^2 + 2x + 4$. If we substitute $x = 10$ (the deci) we get

$$200 + 30 + 4 = 234 .$$

Therefore, the coefficients 2, 3 and 4 are nothing but the digits with appropriate place value decided by the power of x. So far it is easy to see but Vedic Mathematics carries it ahead in a sweeping manner. Consider the polynomial $2x^3 - 3x^2 - 4x + 5$. Apparently, it is difficult to see the clear connection with arithmetic, but after learning the Vedic Vinculum system we can clearly see that this is just another way of writing number $2^{-}3^{-}4\ 5$ (putting $x = 10$). Further, it is not necessary that the coefficients of different terms of x must be all less than the base (here 10). For example

$12x^3 + 11x^2 - 5x + 4$ is equivalent to

$1x^4 + 3x^3 + x^2 - 5x + 4$ which in decimal system is

$12*1000 + 11*100 - 5*10 + 4 = 131\bar{5}4$ also

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$$1*10000 + 3*1000 + 1*100 - 5*10 + 4$$
$$= 131\bar{5}4 = 13054$$

We can further see that x may have any value depending on the unit being used. For example

2 feet 5 inches can also be written as $2x + 5$ (here $x = 12$) ,
2 yard 1 feet can be written as $2x + 1$ (there $x = 3$) .

This opens up tremendous possibilities for furrher simplification and ease in different branches of Mathematics. Some of the interesting applications have already been expounded. (Ref. 1, 3, 4, 5, 10 etc,) Now we can elearly see, how the Vedic procedures of algebra are directly applicable in arithmetic and vice versa. Division by the Paravartaya sutra (No. 4) is one example. Some interesting applications are being presented at other places .

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A NOVEL METHOD OF FACTORIZALION ENVISAGED BY JAGADGURU BHĀRATI KRISHNA TIRTHAJEE MAHARAJ

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According to Mrs Trivedi Jagadguru used to say that He has reconstructed the sixteen formulae from the Atharva Veda, after assiduous research and meditation for eight years. Obviously these formulae are not found in Atharva Vedá in their present form but they have been reconstructed on the basis of intuitive revelation from the materials scattered here and there in the Atharva Vedā. So Vedic Mathematics which was discovered and propounded by late Jagadguru was left in U. S A. in 1958 for publication. But due to ill health as it was not possible for him to go to U. S A. for correctionof proofs and personal supervision of printing the type script was finally brought back to India after this Mahāsamdāhi in 1960. Later on this was published by B. H. U Varānasi in 1965.

In the present article the application of Sutrās of Jagadguru to factorization will be discusesed. There are two sub-sutrās (i) आनुरूप्येण proportionately and ii) आद्यमाद्येनान्त्येमन्त्येन the first by the first and the last by the last.

In a simple case of quadratic expression of the type $ax^2 + bx+c$, split the middle coefficient into two such parts that the ratio of the first ceefficient to that first part is the same as the ratio of that second part to the last coefficient. For example let us take the case $2x^2+5x+2$ in which the middle coefficient is 5 and the parts are (4 and 1), so

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that (2 : 4 = 1 : 2) . Now this ratio *i. e.* ($x+2$) is one of the factors and the second factor is obtained by dividing the first coefficient by the first term of the first factor and the last by the last term. So the second factor is $2x^2/x + 2/2 = (2x+1)$. Again if the parts of the middle coefficients are (1 and 4) we have (2 : 1 = 4 : 2) and hence ($2x+1$) is one of the factors and the second is $2x^2/2x + 2/1 = (x+2)$, Again if we take $2x^2 + 5x-3$, the parts are (6 and 1) so that (2 : 6 = 1 : 3) hence the first factor is ($x+3$) and the second factor is $2x^2/x - 3/3 = (2x-1)$.

The 3rd *S*ubsutra is गुणित समुच्चय: समुच्चय गुणित: which means "The product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product", For example :

$$(x+7)(x+9) = x^2 + 16x + 63$$

$$(1+7)(1+9) = 8 \times 10 = 80 = 1 + 16 + 63 = 80$$

also $(x+1)(x+2)(x+3) = x^2 + 6x^2 + 11x + 6$

$$(1+1)(1+2)(1+3) = 2 \times 3 \times 4 = 24 = 1 + 6 + 11 + 6 = 24.$$

The 4th *S*ub-sutra is लोपन स्थापनाभ्यां which means "by (alternate) Elimination and Retention". Let us consider for example the homogeneous second degree expression .

(1) $E \equiv 2x^2 + 6y^2 + 3z^2 + 7xy + 11yz + 7zx$.

(i) First eliminate z and factorize the remainder by the आद्यम सूत्र

So if $z=0$, $E \equiv 2x^2 + 7xy + 6y^2 = (x+2y)(2x+3y)$

(ii) Similarly if $y=0$, $E \equiv 2x^2 + 7zx + 3z^2 = (x+3z)(2x+z)$

(iii) By the two sets of factors, fill in the gaps caused by making z and y equal to zero So $E \equiv (x+2y+3z)(2x+3y+z)$

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(2) $E \equiv 3x^2 + y^2 - 2z^2 - 4xy + yz - zx$

(i) when $z=0$, $E \equiv 3x^2 - 4xy + y^2 = (x - y)(3x - y)$

(ii) when $y=0$, $E \equiv 3x^2 - zx - 2z^2 = (x - z)(2x + 2z)$

(iii) By filling the gaps we have :

$$E \equiv (x-y-z)(3x-y+2z)$$

(3) $E \equiv 3x^2+xy-2y^2+19zx+28z^2+9xw-30w^2-yz+19wy+46zw$

(i) when $z=0$, $w=0$; $E \equiv 3x^2+xy-2y^2=(x+y)(3x-2y)$

(ii) when $w=0$, $y=0$; $E \equiv 3x^2+19zx+28z^2=(x+4z)(3x+7z)$

(iii) when $x=0$, $w=0$; $E \equiv 28z^2-yz=(7z-2y)(4z-y)$

(iv) when $z=0$, $y=0$; $E \equiv 3x^2+9xw-30w^2=(x-2w)(3x+15w)$

Hence $E \equiv (x+y+4z-2w)(3x-2y+7z+15w)$

(4) $E \equiv 3x^2+7xy+2y^2+11zx+7yz+6z^2+14x+8y+14z+8$

(i) when $y=0=z$; $E \equiv 3x^2+14x+8=(x+4)(3x+2)$

(ii) when $z=0=x$; $E \equiv 2y^2+8y+8=(2y+4)(y+2)$

(iii) when $x=0=y$; $E \equiv 6z^2+14z+8=(2z+2)(3z+4)$

Hene $E \equiv (x+2y+3z+4)(3x+y+2z+2)$.

Now we shall discuss the factorization of cubics and Biquadratics by simple argumentation method. If by any method we are able to know Binomial factor of a cubic we are able to find out quadratic by आद्यमाद्येन... method . An easier derive will facilitated the operation, first and last term are written by आद्यमाद्येन method and the middle term is written by the गुणित समुच्चय ... method for example if we consider the cubic $x^3+6x^2+11x+6$ we see that S_o (sum of the coeffs. of the odd terms) = S_e (sum of the coefficients of the even terms) .

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Hence $(x+1)$ is one of the factors. by आद्यमाद्येन ... method x^3 and 6 are respectively first and last coefficients in the quotient. Also we have S_c (sum of all the coefficients) = 24 and as in $(x+1)$ the $S_c=1+1=2$. we infer that S_c of quotiect is $= 24/2=12$. As the coefficient of the first and last terms are respectively 1 and 6 and their sum is $(1+6) = 7$. the coefficients of the middle term $= 12\text{-}7=5$. S_o the required quotient is x^2+5x+6.

Argumentation Method

We know that $(x+a)(x+b)(x+c) = x^2+x^2\Sigma a+x\Sigma ab+abc$

Example (1) $x^2+6x^2+11x+6$.

Here $S_c=24$ and t_1 (last term) = 6 whose factors are 1, 2, 3 or 1, 1, 6, But sum of the factors must be equal to 6 (the coefficient of x^2) the second group is rejected and the only alternative is to accept 1, 2, 3 group. By testing we see that $\Sigma a + 6$, $\Sigma ab = 11$, and $abc = 6$. Hence $E \equiv (x+1)(x+2)(x+3)$.

(2) $x^2\text{-}6x^2+11x\text{-}6$. Here $S_c=0$ and the last term is -6 whose factors ate -1,-2,-3 . So that $\Sigma a= -6$, $\Sigma ab =11$, $abc = -6$.
Henee $E \equiv (x-1)(x-2)(x-3)$.

(3) $x^2+12x^2+44x+48$. Here $S_c=105$ and $t_1 =48$ whose factors are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 .

Since $S_o \neq S_e$ and $S_c \neq 0$ $(x+1)$ and $(x-1)$ are out of court and possible factors are $x+2$, $x+4$ and $x+6$ as $\Sigma a=2+4+6=12$, $\Sigma ab=8+24+12+44$,

Hence $E \equiv (x+2)(x+4))(x+6)$.

(4) $x^3\text{-}2x^2\text{-}23x+60$, Here $S_c = 36\neq 0$ and $t_1 = 60$ whose factors are 1, 2, 3, 4, 5, 6, 10.12, 15, 20, 30, and 60, But $60= 3\times 4\times 5$

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and when $x=3$, $E\equiv 0$, and when $x=4$, $E\equiv 0$ hence the factors are -3, -4 and 5 as $\Sigma a=-2$, $\Sigma ab=-23$. So

$$E=(x-3)\ (x-4)\ (x+5)$$

(5) $x^4+10x^3+35x^2+50x+24$, Here $S_o=S_e$ hence $x+1$ is a factor. Also $t_1=24$ whose factors are 1, 2, 3, 4, 6, 8, 12, 24 But we must have $\Sigma a=10=1+2+3+4$, also $\Sigma ab=2+6+12+4+8+3=35$ and $\Sigma abc=6+24+12+8=50$

$$\text{Hence } E=(x+1)\ (x+2)\ (x+3)\ (x+4)$$

From above examples we see how powerfully these Vedic sutras given by late Jagadguru, work.

So far as what by I have thought over it, if these Sutrās be pra ctise the mathematicians of the modern world with confidense I can say that they will be quite useful and helpful in solving many other difficult problems of higher mathematics.

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The author expresses his deep sense of gratitude to Dr. S. L. Singh of G. K. University, Hardwar and Dr. R. C. Singh Chandel, the Secretary and Treasurer of "VIJNANA PARISHAD OF INDIA" for inviting me to give an invited talk in the National Symposium on "VEDIC MATHEMATICS, TRADITIONS AND APPLICATIONS" during the First Annual Conference of the Parishad held at G. K. University, Hardwar on March 10-11, 1989,

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VEDIC MATHEMATICS FINDS NO LINK IN VEDAS

By

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No doubt each and every cultured person of the world knows the holy vedas . There are four vedas - Rig, Sama, Yajur and Atharva. The vedic tradition was originally oral. When the vedic period began is uncertain . But, it is expected that this period is 1600 or 1700 B. C. It is learnt that those days the memory of scholars/Rishies was aided by versification and by terse statement of important points they used to say them sutras .

Vedic Mathematics written by late *Śaṅkarāchārya Śri* Bhārati *kṛṣṇa* Tirathaji Mahārāja of Govardhana Matha, Puri is a monumontal work in Mathematics . The sixteen sutras and subsutras propounded by swamiji of course, put forward divine device of calculating arithmetical and algebraical problems mentally . Their neat and ready application to practical problems covers various branches of Mathematics which includes arithmetic, algebra, plane and solid geometry, plane and spherical trigonometry, geometrical and analytical conics, Astronomy, differential and integral calculus etc. The short hand formula in the shape of sixteen sutras and a number of subsutras is a product of rigorus study and assiduous rescarch of swamiji for eight years in the forest surrounding Sringeri . Revcred swamiji used to claim that he had re-constructed the sixteen Mathe matical formulae from the Atharva Veda and its Parisista , But, these formulae are not found in the present recension of Atharva veda .

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Some people are of the opinion that these formulae of vedic Mathematics were actually reconstructed on the basis of intuitive revelation, from materidals scattered here and there in the Atharva Veda.

But, in the daily News Paper "Nav Bharat Times" from Patna on the 26th February 1988 there was an Article written by Sri Gunakar Mule on vedic Mathematics . In that Article he had tried his best to prove that with a view to make vedas popular the Sixteen Vedic Sutras had been written . We find that he is very much firm in his opinion that the sixteen sutras and subsutras of Vedic mathematics are, of course, mental creation of Swamiji and they have nothing to do with Vedas . He quotes Sri V. S. Agrawala's assertion in his General Editor's foreword of vedic Mathematics that the Sutras and Subsutras do not appear in the hither to known Parisista of Atharva Veda. Mr. Agrawala further admits that the style of language also points to their discovery by swamiji himself . It is obvlous that none of renowned Hindu Mathematicians Aryabhatha, Brahmgupta, Mahavirachary and Bhaskaracharya who were also known to be well-versed in vedas could not point out that vedas are rich in Mathematics . Mr. Mule feels that Swamiji has tried to establish the supremacy of Vedas in the field of Mathematics too by tagging his own creation in the name of Vedas .

I myself have gone through the four published Vedas-Rig, Same, Yajur and Atharva and their Parisistas . But, I have also failed to find any trace of the sixteen sutras and their corollaries of the Vedic Mathematics in any form and in any context in any of these vedas and their Parisistas . And, henee it is a duty to we people to trace the sixteen Vedic Sutras in the Vedas .

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COMPUTATIONAL ASPECTS OF NONLINEAR OPTIMIZATION PROBLEMS AND THEIR PRACTICAL APPLICATIONS

By

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ABSTRACT

This paper surveys in brief the available literature on optimization and tries to highlight the problems being currently faced in the numerical solution of nonlinear optimization problems. Practical use of nonlinear optimization techniques in solving real life problems arising in different fields is also briefly discussed.

1. INTRODUCTION

Mathematical techniques for finding the optimal value (the greatest possible value or the least possible value) of a function are called 'optimization technique's. The function to be optimized (maximized or minimized) may be a function of any number of independent variables. It may also be subject to certain constraints in the form of inequalities or equalities. Sometimes the number of functions to be optimized may be more than one.

Optimization problems arise in various disciplines such as engineering design, manufacturing systems, economics, business administration, physical sciences etc. In fact the newly developed optimization techniques are now being tried in every sphere of human activity where decisions have to be taken in some complex situations that can be represented by maethematical models.

Keeping in view practical applications, efforts have always been

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there to develop simple, efficient, and reliable methods for solving different types of optimization problems. Methods of calculus for determining the maxima and minima of functions were developed to help solve real life practical optimization problems . However, it was soon realised that the utility of the methods of calculus in finding the maxima or minima of a function gets very much restricted when the function is a function of several unknown variables and is subject to a number of constraints in the form of inequalities, or when the function to be optimized is nondifferentiable . Since the mathematical models of a wide variety of practical problems are of this nature, a necessity was felt for developing computatlional techniqes which are specifically suited for solving such types of nonlinear optimization problems . This lead to the systematic development of a subject which is now commonly known as 'Mathematical Programming' or 'Optimization Techniques' . The advent of high speed computing machines and the growing requirements of society for the optimal economic and efficient use of the available natural resources has given a further fillip to the development of this subject . Currently a vast amount of literature is available on the subject. However the available techniques are far from being adequate in helping efficiently solve a wide variety of practical real life problems which are being thrown up by the rapid technological, social and industrial developments which are taking place daily. Efforts are, therefore, being continuously made to develop more efficient, reliable and faster methods for solving such practical problems .

In this paper we have tried to briefly acquaint the reader with the current state of affairs and some of the possible emerging trends in this field .

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2. SOME BASIC DEFINITIONS

Different types of practical problems lead to different types of methematical models for which optimal solutions have to be obtained. However the most general form of a single objective optimization problem is :

Maximize (or Minimize) $f(X)$, $X \equiv (x_1, x_2, \ldots, x_n)$ **(2.1)**

Subject to $g_j(X) \leqslant$ or $=$ or $\geqslant b_j$, $j = 1, 2, \ldots, m$ (2.2)

$x_1, x_2, \ldots, x_n$ are called 'unknown variables' or 'decision variables' and $X \equiv (x_1, x_2, \ldots, x_n)$ is called a 'decision vector'. The function (2.1) which is to be maximized or minimized is known as the 'objective function'. The set of inequalities (2. 2) constitute the set of 'constraints' of the problem The optimization problem (2.1 – 2.2) has n unknowns and m constraints, By the solution of the optimization problem we mean the set of values of the unknowns $x_1, x_2, \ldots, x_n$ which do not violate the constraints (2. 2) and at the same time optimize the objective function (2, 1).

The branch of mathematics concerned with solving such type of optimization problems is known as 'mathematical programming'. If, in an optimization problem the objective function $f(X)$ as well as the constraint functions $g_j(X)$ all happen to be linear then it is called a 'linear programming problem (LPP)'. If the objective function and (or even one of the constraints is nonlinear, it is known as a 'nonlinear programming problem (NLPP)'. An optimization problem in which the objective function is not subject to any constraints is called an 'unconstrained optimization problem' otherwise it is called a 'constrained optimization problem'. An optimization problem is called an 'integer (all integer) programming problem' if all the

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variables $x_1, x_2, \ldots, x_n$ are restricted to have integer values. It is called a 'mixed integer programming problem' when some of the variables are restricted to have integer values while integer restrictions are not imposed on other variables. An optimization problem in which the variables are allowed to have either zero or one value is called a 'zero-one programming problem'. Problems in which the number of objective functions (2.1) to be optimized is more than one are called 'multiobjective problems'.

The set S of points which satisfies all the constraints of an optimization problem is called a 'feasible set' or a 'feasible solution'. In a minimization problem if for $X^* \epsilon S$ there exists neighbourhood $N\epsilon (X^*)$ around X^* such that $f(X) \geqslant f(X^*)$, $X \epsilon S \cap N\epsilon (X^*)$, then X^* is called a point of local optimality (or 'local optimal solution'). If $X^* \epsilon S$ and $f(X) \geqslant f(X^*)$, $X \epsilon S$, then X^* is called a 'global optimal solution',

A nonlinear optimization problem may have one or more than one local optimal solutions. If the problem has a unique local optimal solution then it is also the global optimal solution. However, if the problem has more than one local optimal solutions then one or more of these may be global optimal solutions. In a linear programming problem a local optimal solution is also the global optimal solution. However, in a nonlinear optimization problem this is not always the case. A nonlinear optimization problem in which the local optimal solution is also the global optimal solution is called a 'convex programming problem'.

3. LINEAR PROGRAMMING PROBLEMS (LPP)

Linear programming is at present the most completely developed technique of nnmerical optimization. Linear programming techniques

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are being widely used to solve practical problems in varieiy of fields such as economics, industry, transportation, social sciences, etc. A number of standard texts are available on this subject (see for instance Gass [47]). Simplex method developed by Dantzig [27] is still the most commonly used method to solve linear programming problems. Many practical problems formulated as linear programming problems have hundreds of constraints and huudreds of decision variables. These invariably have to be solved using digital computers. The first successful attempt on a first generation computer was made at National Bureau of Standards in early 1952. Further refinements in the simplex method were consequently carried out so that the implementation of the simplex method on a computer could be done more efficiently. Several versions of the simplex method are now available which can efficiently handle large size linear programming problems on computers. In 1984 Karmarkar [72] designed projective algorithm which can efficiently handle large size linear programming problems consisting of several thousand unknown variables and several thousand constraints.

Methods for solving special types of linear programming problems have also been developed. An important class of linear programming problems is called 'transportation or distribution problem'. The term 'integer programmiag' refers to the class of linear programming problems where some or all of the decision variables are restricted to be integers. Important developments in the earaly stages of the theory of LP are given in Dantzig [27]. Gale [46] and Hadley [57] give a mathematical treatment of simplex method from the view point of vector space theory. An excellent bibliography, on the applications of LP in practice is available in Gass [47]. Gale [46] discusses many linear programming models of production, exchange and matrix games

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4. NONLINEAR PROGRAMMING PROBLEMS (NLP)

Whereas theoretically, every LPP can be solved by simplex method, there is on single technique which can claim to efficiiently solve each and every nonlinear optimization problem. In fact, a technique which is efficient for one nonlinear optimization problem may be highly inefficient for solving another nonlinear optimization problem. A variety of computational techniques have, therefore, appeared in literature for solving nonlinear optimization problems (see for instance Himmelblau [61], Bazaraa and Shetty [7], McCormick [84]) Most of these, however, determine at the most a local optimal solution. In the next few section we discuss in brief some of the important methods for solving nonlinear optimization problems which are avaiable in literature.

5. METHODS FOR SOLVING UNCONSTRAINED NONLINEAR PROGRAMMING PROBLEMS

Unconstrained opiimization deals with the problem of minimizing (or maximizing) a nonlinear function of one or more more independent variables in the absence of any constraints. Several iterative procedures are available in literature for solving an unconstrained optimization problems. Most of these procedures involve a line search. In general, the efficiency of the line search method greatly affects the overall performance of the solution technique. For optimization of a nonlinear function of one variable some of the commonly used techniques are: the Fibonacci search procedure credited to Kiefer [71], the golden section method and bisection method etc. These procedures use only function evaluations at certain points in the region of search. Another class of procedures for the unconstrained optimizatio problems use curve fitting methods. If a function of one variable is to be minimized, the procedure involves finding an

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approximating quadratic or cubic function and then minimizing it. Some limited computational studies on the efficiency of this approach are given in Himmelblau [61].

Among gradient free methods, the method of Rosenbrock [101] and the method Zangwill [123 (a)] are generally considered quite efficient for solving unconstrained problems of several variables. As originally proposed the Rosenbrock [101] method and the procedure of Hooke and Jeeves [64] do not use line search but employ instead discrete steps along the search directions. Incorporating a line search within the Rosenbrock's procedure was suggested by Davies, Swann, and Compey and is discussed by Swann [110].

There are yet other derivative free methods for unconstrained minimization of functions of several variables A procedure that is distinctly different is called the sequential simplex method. The method was proposed by Spendley. Hext and Himsworth [108] and modified by Nelder and Mead [90]. The method essentially looks at the functional values at the extreme points of a simplex. The worst extreme point is rejected and replaced by a new point along the line joining this point and the centroid of the remaining points. The process is repeated until a suitable termination criterion is satisfied.

The method of steepest descent proposed by Cauchy in the middle of the nineteenth century continues to be the basis of several gradient based solution procedures. The method of steepest descent uses first order approximation of the function being minimized and usually performs poorly as the optimum is reched. On the other hand, Newton's method uses second order approximation and usually performs well at points close to the optimal. Several variations of Newton's method are also available. Recently Coleman and Conn

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[25] have proposed some methods based on the second order derivative information .

Among the unconstrained optimization techniques, methods using conjugate directions are considered efficient . For a quadratic function these methods give the optimal solution in at the most *n* steps, *n* being the number of unknown independent variables in the problem. Some of the commonly used methods of this category are Powell [97*b*] and Zangwill [123*a*] . Some of the recent techniques which have appeared in this category are Dixon et al. [34], and Axeleson [6] .

Another class of methods, termed as vaiiable metric or quasi-Newton methods, approximates the hessian matrix or its inverse using information from only first order derivatives . One of the early methods of minimizing a nonlinear function using this approach is that of Davidon [28], which was simplified and reformulated by Fletcher and Powell [41] and is referred to as the variable metric method . A useful generalization of the Davidon-Fletcher-Powell method was proposed by Broyden [19]. Further details about variables metric methods are avilable in Spedicto [107] . For further details on methods for unconstrained nonlinear optimization one may refer to standard texts on the snbject such as Avriel [5], Bazaraa and Shetty [7] and McCormick [84] etc .

6. METHODS FOR CONSTRAINED NONLINEAR PROGRAMMING PROBLEMS

Several authors have tried to use unconstrained optimization methods for solving nonlinear problems with constraints . A successsful and frequently used approach is to define an auxilliary unconstrained problem such that the solution of the unconstrained

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problem yields the solution of the constrained problem. Some authors have also modified the unconstrained optimization methods to handle constraints. Goldfarb [55] has extended the Davidon-Fletcher-Powell method to handle problems with linear constraints utilizing the concept of gradient projection. The method was generalized by Davies [29] to handle nonlinear constraints. Gill and Murray [50] have presented a survey on Newton type and Quasi Newton type methods for solving constrained problems.

Several gradient free search methods have also been extended to handle linear and nonlinear constrained problems Glass and Cooper [52] extended the method of Hooke and Jeeves [64] to deal with constraints. Box [15] developed a constrained version of the simplex method. Other alternative versions of the method were developed by Ghani [48], Umida and Ichikawa [115]. Another method that uses the simplex technique in constrained optimization was proposed by Dixon [33].

Another important class of methods used for solving constrained nonliear programming problems are known as penalty function methods. In penalty function methods the constrained problem is converted into an unconstrained problem or a sequence of unconsrtained problems in which there is a severe penalty for the violation of constraints. Use of penalty functions is generally attributed to Courant. However, significant progress in solving practical problems by the use of panalty methods followed the classic work of Fiacco and McCormick [38*a*].

Several extensions of the concept of penalty function have also been made. The work in this direction is still in progress (The

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interested reader may refer to Fiacco and McCormick [38 *b*], Zangwill [123 *b*]).

Another class of methods for solving nonlinear optimizetion problems is the family of exact penalty function methods. The pioneering work in this field has been done by McCormick [84]. Work in this direction has also been done by Fletcher [40] and others. Recently Pillo and Grippo [96] have proposed a continuously differentiable penalty function for nonlinear programming problems.

Yet another class of methods used for solving constrained nonlinear optimization problems are known as methods of feasible directions. The method of feasible directions is a general concept that is exploited by a primal algorithm that proceeds from one feasible solution to another. One of the well known methods of this class is the feasible direction method of Zoutendijk [127]. Zangwill [123*b*] presented an improved version of a convergent algorithm based on the use of the concept of near-binding constraints In 1960 Rosen developed the gradient projection method for linear constraints and later in 1961 generalized it for nonlinear constraints.

The method of reduced gradients was developed by Wolfe [119]. It was later generalized for nonlinear constraints by Abadie and Carpentier [1] who considered several approaches. Zangwill proposed the convex simplex method for solving a nonlinear programming problem with lenear constraints. The method could be viewed as a reduced gradient method. A comparison of convex simplex method with the reduced gradient method is given by Hans and Zangwill [59]. Recently Cheng [23] has given a dual gradient method for linearly constrained problems.

Keefer [70] proposed a method in which the basic search is the

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Nelder and Mead simplex technique. The lower and upper bound constraints on the variables are dealt with explicity, while other constraints are handled by a penalty function scheme.

Paviani and Himmelblau [95] also use the simplex method in conjunction with a penalty function to handle constrained problems. The method is also known as flexible tolerance method. Flexible Tolerance method developed by Paviani and Himmelblau [95] uses only function evaluations to compute the optimal solutions of constrained as well as unconstrained optimization problrms. It combines the method of Nelder and Mead [90] with various additional rules to take care of equality and/or inequality constraints. The set of violated inequalities and equalities is lumped into one inequality constraint loosely satisfied during its initial stages. As the seareh proceeds the near feasibility iimits are gradually made more and more restrictive until an acceptable feasible vector is obtained. In case of nonconvex problems the method eusures only a local optimal solution.

The sequential linear programming (SLP) approach for constrained oytimization problems finds the optimal solution by repeated linear approximation of a nonlinear problem and using linear programming techniques to solve it. These techniques are discussed in detail in McCormick 84] and Himmelblau [61]. Similarly, in sequential quadratic programming the basic concept is to create sequential quadratic approximations of the original nonlinear optimization problem. (For further details see for instance Biggs [11]. The latest discussions on this topic may de found in Conn [26]).

Sequential convex linearization is another approach [115] which has been tried. The basic concept of this approach is to first linearize the objective and the constraints as is done in SLP, However,

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now instead of solving this line arized problem reciprocal variables are used to create a conservative convex approximation .

Attempts have also been made in literature to obtain the solutions of both constrained and unconstrained optimization problems by solving the Kuhn-Tucker conditions directly . Methods under this category are some times classified as multiplier methods (see for instance Arrow and Hurwitze [4]). Quasi-Newton type methods have been used in literature to obtain the solution to the systems of equations representing the Kuhn-Tucker conditions [13, 97*a*] . Mohan and Tahir [87] also tried to obtain solutions to constrained and unconstrained nonlinear optimization problems by using a least square algorithm for solving a system of nonlinear equations representing the Kuhn-Tucker conditions of the problem .

Several studies for evaluating and testing nonlinear programming algorithms have been made . Computational results are reported in Himmelblau [61] using a range of nonlinear programming problems with varying degree of difficulty including highly nonlinear constraints and objective functions . For further details the interested reader can refer to Ragsdell 99] and Hung [65] .

A class of nonlinear optimization problems which occurs frequently in problems of engineering design and which has been studied extensively in literature for quite sometime now is known as 'geometric programming problems' (see for instance Duffin, Peterson and Zener [36] and Beightler and Phillips [8]) . The concept of geometric programming problems was later generalized to include the cases in which some of the cost coefficients may also be negative . Such problems are now known in literature as 'generalized geometric programming problems' (GGPP) or 'signomial geometric programming

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problems, (SGPP). (c. f. Beightler and Phillips [8]). The geometric programming problems and signomial geometric programming problems are often solved using duality approach (see for instance Dembo [30], Beightler and Phillps [8]). Besides duality approach some other approaches have also been tried (for instance Mohan and Shashi Tyagi [88]).

In 1978 Lidor and Wilde [81] and Wilde [118] extended the concept of geometric programming problems to a more general class of nonlinear optimization problems which they called 'transcendental programming problems' (TPP) The concept of TPP was extended by Veena [116] to include the problems in which all the coefficients need not be positive and called these 'Signomial Transcendental Programming Problems' (STPP).

Several methods have also appeared in literature which are specifically suited for solving special types of nonlinear optimization problems. Mention in this regard may be made of the methods for solving Quadratic programming, Geometric programming, Separable programming, Dynamic programming, and Fractional programming problems. Discussion on these topics is available in standard texts such as Himmalblau [61], Beightler and Phillips [8]. Bazaraa and Shetty [7], McCormick [84] and Hartley [60] etc.

7. METHODS FOR DETERMINING THE GLOBAL OPTIMAL SOLUTION

In the absence of convexity, most of the methods discussed above at the most guarantee a local optimal solution. Keeping in view the practical necessity and the availability of fast computing machines, and computational techniques are now being reported in literature which try to search for the global optimal solution of nonlinear

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optimization problems. Whereas Dixon and Szego [35*b*] give the status of the subject till seventies, Boggs [13] updates it to early eighties

The methods currently available in literature for solving global optimization problems may be broadly classified as deterministic methods and probabilistic methods. The deterministic methods try to guarantee that a neighbourhood of the global optima is attained. Such methods do not use any stochastic techniques, but rely on a thorough search of the feasible domain. They are applicable, however, to a restricted class of functions only such as, Lipschitz continuous functions, one dimensional functions, etc.

The trajectory method of Bramin [17] is one of the earliest deterministic methods. Treccani [114] showed that trajectory methods fail on some functions. Moreover these methods need analytic derivatives and often prove very expensive to use. Recently, Diener [32] proposed a new trajectory method but this too does not guarantee the global solution. Evtushenko [37] and Shubert [105] proposed grid methods for functions satisfying a Lipschitz condition. But these methods are not practical because it is not easy to find Lipschitz constant. Levy et al. [78*a*, 79] proposed a tunnelling algorithm for twice differentiable functions. Although the method generally succeeds in locating the global solution, it does not guarantee whether the global solution has been really found or not. In 1985 Levy and Gomez [78*b*] modified the Tunnelling algorithm and called it spherical tunnelling algorithm whidh they claim is better than the previous version.

In stochastic or probabilistic methods two phases are generally employed. In the first phase, or the global phase, the function is

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evaluated at a number of randomly sampled points. In the second phase, also called the local phase, the points are manipulated by local searches to yield a possible candidate for the global minima. Although probabilistic methods do not give absolute guarantee of success, these methods are sometimes preferred over the deterministic methods because they are applicable to a wider class of functions as they depend upon function evaluations alone and do not require the computation of derivatives etc.

Some of the earlier methods of this category are pure random search methods by Anderson [3], Brooks [18] and Palosarri et al. [93]. Torn [113] proposed a clustering algorithm which was tested on a number of problems by Betro [9]. Boender and Rinnooy Kan [12] use Bayesian approach to solve global optimization problems. Dixon et al. [35*b*] suggested a method in which any local search method could be used to search the global solution by starting repeatedly from different initial points chosen stochastically.

Price [98] presented a controlled random search method in which the simplex approach is used on a random sample of points to yield a better point at each iteration and the method in the limit converges to the point of global minima. Recently, Mohan and Kusum Shanker [86] have updated the random search method of Price [98*b*] for solving the constrained nonlinear optimization problems. Shashi Tyagi [104] has developed a suitably modified version of random search method to solve integer and mixed integer programming problems.

8. MULTIOBJECTIVE AND GOAL PROGRAMMING PROBLEMS

Optimization problems in which it is desired to optimize more than one objective are called 'multiobjective' or 'vector optimization

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problems'. Mathematical model of a multiobjective vector optimization problem is :

$$\text{Maximize or Minimize } [\, f_1\ (X), f_2\,(X)\,, \ldots , f_k\,(X)\,]^T \quad (8.1)$$

$$\text{Subject } g_j\,(X) \leqslant b_j\,,\ j=1, 2\,, \cdots, m \quad (8.2)$$

Mathematical models of many real life optimization problems turn out to be multiobjective problems and these are now being extensively studied in literature (see for instance Zeleny [124], Changkong and Haimes [21], Steuer [109] and Zionts [126] etc.). Multiobjective problems arise in different fields such as Game Theory, Networks, Markov Processes and engineering design (see for instance Golabi [54], Thomas [112 and Yu [120]).

Many methods have been proposed in literature for solving multiobjective problems. Surveys and overview of multiobjective programming methods are given in Cohon et al. [24], Ignizio [66*b*] and Zionts [126].

In a goal programming problem, it is desired to make the objective function achieve a specified value (called goal) as closely as possible rather than obtaining its optimal value. A goal programming problem may have a single objective or multiple objectives for whieh the goal/goals to be achieved are specified.

Goal programming was first used for linear problems by Charnes and Cooper [22*a*, and extended by Iziri [68] and Lee [76]. In fact as they (Charnes and Cooper [22*b*]) point out later, they developed goal programming in the context of eonsttained regression when they attempted to find a formula for executive compensation in a division of the General Electric Company. There havs been numerous subsequent works on goal programming, particularly in the areas of

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algorithmic development, applications and extensions (see for instance the survey by Karnbluth [73], texts on goal programming by Lee [76] and by Ignizio [66*a*] other references therein) .

Fuzzy set approach is now also being used in solving goal programming problems . In 1965 Zadeh [121] provided the first tool (fuzzy set) specially devised for dealing with different forms of imprecision, vagueness . By now over two thousand works dealing with this topic have been published. The theoey that Zadeh called fuzziness also stimulated a constructive debate on the several forms of uncertainty (Gaines [45]), and their mathematical rapresentation (Hohle and Klement [62] etc .).

Several authors kave used fuzzy set theory in solving goal programming problems . (see for instance Zimmermann [125], Hannan [58], Kacprzyk [69]) . They demonstrate how fuzzy or imprecise aspirations of the decision maker can quantified through the use of piecewise linear continuous functions. Shashi Tyagi [104] considered the effectiveness of solving multiobjective and goal programming problems using random search technique of global optimization as developed by Mohan and Kusum Shanker [86] .

9. SOLUTION OF PRACTICAL REAL LIFE PROBLEMS

Optimization teehniques, linear as well as nonlinear for single as weli as multiobjective peoblems, find extensive practical use in various disciplines such as engineering design economics, business administration, physical sciences and manufacuring systems etc (see for instance Bracken and McCormick [16], Beightler and Phillips [8], Himmelblau [61], Wilde [118], Osyczka [91] and Ozan [92] etc). The work in this field is interdisciplinary in nature . In this section

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we present a brief survey of available literature on topics in which we have been primarily working .

Gasoline blending problem, Circuit design problem, Foundry charging problem and Tolerancing mechanism components problems are solved using linear programming methods (c f. Oxan [92]). Mohan and Kusum Shanker [86] optimized the reliability of a complex system using random search techniqus , Paul and Tay [94] determined the optimal design of an Industrial refrigeration system . Sophos et al . [106] used goal programming in modeling petrochemical industry. Zanakis et ai . [122] solved chemical production planning problem via goal programming approach. Sakawa [102] solved multiobjective reliability design of a stand by system using sequential proxy optimization technique (SPOT) Grauer et al . [56] applied multiobjective criteria in chemical engineering . McCommon [83] apdlied goal programming approach in the design of Electro-acoustic transducers and transducer array . Nakayama et al . [89] used trade-off method in structural design .

Nonlinear optimization techniques have also been used for inversion of gravity data in geophysical prospecting (Al Chalabi [2], Fisher and Howard [39], Last and Kubik [75], Kusum Shanker, Khatri and Mohan [74*b*] . The problem of determining the location and the time of origin of an earthquake has also been solved as a nonlinear optimization problem (Gaiger [43], Bolt [14], Flinn [42], Gill and Murray [49], Dennis et al . [31], Gill et al. [51], Hong and Bond [63], and Kusum Shanker [74*a*]) .

Nonlinear optimization techniques also find extensive applications in structural engineering optimization. Venderplaats [117] considers the use of Numerical Optimization Techniques in Computer Aided

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Optimal Design and Mechanical Systems. Minimum cost design of reinforced overhead water tanks has been considered by Rao and Raghavan [100], Chand et al. [20], Saxena et al. [103*a*, 103*b*], Templeman [111] optimized highway bridges using geometric programming. Saxena et al [103*c*] used nonlinear technigues of global optimization to obtain the minimum cost designs of the gas holders of biogas plants . Optimum designs of grid floor structures have been obtaincd by Goel [53 . Iyenger and Gupta [67] and Manu Saxena [82]. Manu Saxena [82] also used random search based technique of global optimization to obtain the minimum cost designs of reinforced concrete hanger type structures for storage godowns, industrial seeds and food grain storage silos. Some books on structural engineering have also appeared in literature. A comprehensive survey of structural engineering optimization techniques is available in Lev [77] . Levy and Lev [80] have reviewed recent developments in structural engineering optimization. Gaindhar, Mohan and Shashi Tyagi [44] have ussd nonlinear optimization methods to optimize riser design in metal casting. They have also considered the problem of determining the values of design parameters which can obtain desired values of characteristics in the mould in vacuum moulding process as a multiobjective noninear optimization problem [85] .

Bharati et al. [10] have used nonlinear optimization techniques to analyse the problem of town planning wiih particular reference to the prcblem of allocating the excess population of Delhi to various other towns of the National Capital Region .

10. EMERGING TRENDS

In view of the numerous practical applications of the optimization techniques and the fact that these can go a long way in proper

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planning of production and efficient utilization of the available natural resources to meet the growing aspirations of the society, work is going on in this field at a rapid pace. Even though quite a large number of present day real life practical problems can be solved using the currently available techniques of optimization, there are still a large number of real life problems which cnnnot be satisfactorily solved (and some times even not solved) by the currently available techniques. Most of the currently available techniques of nonlinear optimization can efficiently solve moderately sized nonlinear optimization problems on large mainframe computers. Eventhough linear programmiug problems having several thousand unknown variables and constraints can be solved efficiently, there is on guarantee that a nonlinear optimization problem consisting of even a hundred or so unknown variables and constraints will be satisfactorily solved by available algorithms even on main frame computers. Efforts therefore being currently made to develop algorirhms which can efficiantly solve moderate sized nonlinear optimization problems on personal computers (PC's) since practical users who are faced with moderate sized nonlinear optimization problems can at the moment ouly couveniently afford the facilities of PC's and not of large size compuing syftems Simultaneously efforts are also being made to develop algorithms which can solve large size nonlinear optimization problems on mainframe computers. Currently with the introduction of Super eomputers in which parallel processing is possible, efforts are also being made to develop algorithms which make use of this facilitp. Efforts are also being made to develop algorithms which can solve nonlinear multiobjective goal programming problems in a more realistic and efficient manner as these problems more appropriately reflect the realities of true life situations.

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11. CONCLUDING OBSERVATIONS

In this paper an effort has been made to briefly acquaint the reader with the current state of affairs in the field of nonlinear optimization techniques and their use in solving practical real life problems. However it is in no way claimed that the survey is exhaustive. There are several important methods and several important practical applications which have not been referred here. The interested reader can find reference to many such important works in the books and bibliographies of research papers given in references at the end of this paper.

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MATHEMATICAL AND COMPUTER BASED NUMERICAL APPROACH TO PHYSIOLOGICAL HEAT TRANSFER PROBLEMS

By

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1 INTRODUCTION

All living animals produce heat, which is either migrated to the surroundings or stored in the body itself. All mammals and most birds use this heat to maintain the internal parts of body at constant temperature. Heat is continuously being generated within the body as a result of chemical processes associated with breathing, digestion and physical efforts. The most important factor responsible for heat generation in a human body is metaboliam. The metabolic reactions take place in body cells. The rate of metabolic heat generation usually depends on environmental conditions.

Heat removal from an animal body is also a continuous process like heat generation. But in heat generation almost all regions of the body are involved while the migration of heat to the surroundings takes place only through the skin and respiration. However, the heat lost through respiration is not much and the quantitiy lost is easily made up by the dense system of blood capillaries present in the pulmonary region. The exchange of heat through peripheral regions is controlled by various body organs and physical conditions. The complexity of region makes this process very interesting and most of the physiological heat transfer problems are concerning with heat transfer through skin and subcutaneous tissues (SST). Any unified study of such problems has to take into account various factors

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like metabolic heat generation, micro-circulation, perspiration, structure of the region with its complex physical parameters and environmental conditions.

Till recently, there has been hardly any analytical study of physiological heat transfer problems with an idea of providing general useful relationships. Some initial and specialized studies in allied aspects include the works of W. perl (J. Theo. Biol..2 (1962), 201), G. J. Trezek and T. E. Cooper (Therm. Prob. Bio. Tech. ASME, 1968, 1), V. P. Saxena (Proc. VI Heat Tr. Conf. 3 (1978). 65) and J. Stekete and M. J. Van Der Hoek (Phys Med. Biol., 24 (1979), 31). Some papers on heat transfer problems in SST region of human body for some simple and moderate conditions have been brought out by V. P Saxena (Proc. XI Nat. Conf. Fl. Mech. Fl. Power, 2 (1979), 156), V. P. Saxena and D. Arya (Num. Methods in Thermal problems, Editor : Morgan and Lewis, Pineridge Press U. K.), V, P. Saxena and D. Arya (J. Theo. Biol., 81 (1981), 423), V. P. Saxena and J. S. Bindra (Proc. Symp. Math. Mod. M. R. I. Alla. (1982)), D. Arya and V.P. Sexena (Proc. Symp. Math. Mod. M. R. I. Alla. (1982).

Some recent work by Saxena (J. Theo. Biol. 102 (1983), 277) and Saxena and Bindra (Ind. J. Pure and Applied Math. 15 (1984), 31, 18 (1987), 846) presents total study of normal heat flow in dermal layers by exact as well as numerical approach. Heat regulation in body limbs is already under study at present, by Saxena and Bindra (Int. J. Math. and Math. Sci. (1989)).

2. Mathematical Formulation.

The following equation governing heat transfer in skin and subcutaneous tissues has been obtained by making use of Fourier's laws and Fick's well known principle.

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$$div\ [\ K(\bar{\bar{x}}, u)\ \text{grad}\ u(\bar{\bar{x}}, t)\ ] + m(\bar{\bar{x}}, t)^{C_b}\ \{u_b(\bar{x}, t) - u(\bar{x}, t)\}$$

$$+ S(\bar{\bar{x}}, u) = P(x)\ C(\bar{x})\ \frac{\partial u}{\partial t}(x, t) \qquad ...(1)$$

and $K\frac{\partial u}{\partial n} = h(u - u_a) + LE$ ..(2)

at the skin surface

where

$K=$ thermal conductivity of the tissue

$m=$ mnss blood flow rate

$c_b=$ specific heat of blood

u_b = blood temoerature

$S=$ rate of metabolic heat generation

$p=$ tissue density,

$c=$ specific heat of the tissue

$u(\bar{x}, t)$ = temperature of the tissue at any position $\bar{x}$ and time t,

$\bar{x}=$ space variable,

$t=$ time variable,

$h=$ heat transfer coefficient

$u_a=$ atmospheric temperature

$L=$ latent heat of evaporation

$E=$ rate of evaporation

$\frac{\partial u}{\partial n}$ = partial derivative of u along normal to the surface .

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The terms on the left of (1) are Fickian diffusion, perfusion and metabolic heat generation respectively Various authors have used special cases of the above equations . Due to composite behaviour of the medium and irregular geometry of the subregions we have consid-red following variational form of the governing equations for heat flow in dermal and other peripheral parts of a human body :

$$I = \frac{1}{2} \int_{P} [\, k_x (u, x)^2 + k_y (u, y)^2 + k_z (u, z)^2$$

$$+ M c_b (u_b - u)^2 - 2Su \,] \, dP$$

$$+ \frac{1}{2} \int_{Q} [\, h (u - u_a) + 2 \, LEu \,] \, dQ$$

where Q is the boundary of the region P under consideration . This region can be divided into natural and convenient subregions depending on the variation of vorious parameters like tissue structure, tissue thermal properties, blood mass flow rate and rate of metabolic heat generation. The variational finite element method provides flexibility of local considerations . In most our problems we have exploited this advantage of FEM and obtained solutions under various practical situations.

The same model in polar cylindrical co-ordinates has been employed to human limbs. However, to avoid cumber some computation and to save computer time we have modified FRM approach and deviced, what we call as Psuedo Analytic Finite Partition Method for circular cross sections. Assuming annular layered structure of the limbs and unsymmetical situation of blood vessels, the FRM technique has been used only in radial direction. The resulting system of differential

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equations have been solved by using analytical methods such as Fourier series and Laplace transforms .

Recently we have taken up work related to abnormal physiological situations . At present our main attension is on the growth of malignant tumours in dermal regions and its effect on surface and internal temperntures. We have incorporated the effect of ever increasing uncontrolled metabolic activity of the tumour and changing profile of blood perfusion. This includes the nacrosis taking place at the core (Saxena and Pardasani; Int.Conf. Biomath. Xian. China 1988). (Pardasani and Saxena : Biomechanics, ed by Sahay and Saxena, published by Wiley Eastern, New-Delhi. 1988),for two dimensional traigular shape functions approximating main and tumour parts. A computer software has been developed for the same and ICIM-6000 machine has been used to obtain numerical results.

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AREA OF POLYGON

By

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1. Area in Terms of Sides of Polygon:

$$\text{Area} = \frac{na^2}{4} \cot\frac{\pi}{n}$$

where a is the side of polygon of n sides.

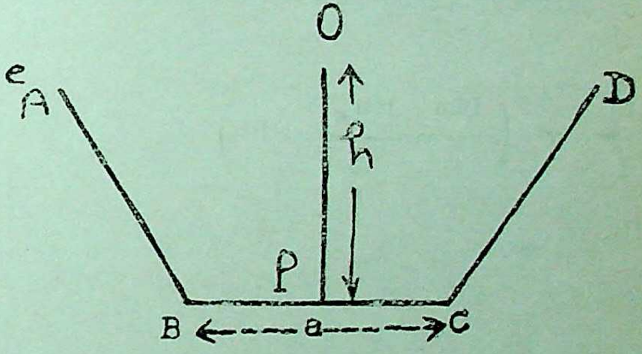
Derivation: Let ABCD......be a polygon of n sides with side length a. Let O be the centre of the polygon. Let h be the length of the perpendicular from O on one side (Fig 1). Now

$$\angle \text{BOP} = \frac{\pi}{n}.$$

From $\triangle$ BOP we have

h = BP Cot BOP

$$= \frac{a}{2}\text{Cot}\ \frac{\pi}{n}.$$



Therefore area of $\triangle$ AOC

$$= \frac{1}{2}a \times \frac{a}{2}\ \text{Cot}\ \frac{\pi}{n}$$

$$= \frac{a^2}{4}\ \text{Cot}\ \frac{\pi}{n}.$$

Hence Area of Polygon

$$= \frac{na^2}{4}\ \text{Cot}\ \frac{\pi}{n} \qquad \text{......} (1)$$

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2. Area in Terms of h.

From $\triangle$ BOP

$$\frac{a}{2} = h \tan \frac{\pi}{n}$$

or $$a = 2h \tan \frac{\pi}{n} \quad .$$

Putting this value of a in (1) we have

$$\text{Area of Polygon} = nh^2 \tan \frac{\pi}{n} \quad \cdots\cdots \ (2)$$

3. Deduction of Formula for Area of Circle:

If we take $n \to \infty$ then polygon will become circle. In this case $\lim\limits_{n\to\infty} h = r$ (the radius of circle).

Thus area of circle

$$= \lim_{n \to \infty} nh^2 \tan \frac{\pi}{n}$$

$$= \lim_{n\to\infty} \frac{\pi h^2 \tan \frac{\pi}{n}}{\frac{\pi}{n}}$$

$$= \pi r^2 \quad \left(\lim_{n\to\infty} \frac{\tan \frac{\pi}{n}}{\frac{\pi}{n}} = 1 \right) \quad .$$

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MATHEMATICS AND ITS APPROACH TO TECHNOLOGY

By

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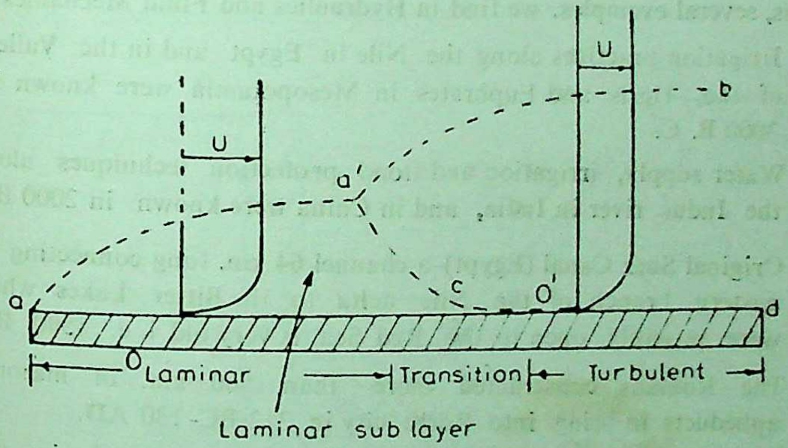
Mathematics as we study from the lower level to the higher level, changes its nature of study, apalications etc. For example in elementary Algebra, we solve some equations say

$$3x+y = 2$$
$$x+4y = 8$$

and get $x=0,\ y=2$

Now see the use of this simple Algebra in solving the equations of straight lines for the Coordinates of the points of intersections. This we study in early stage and it is said to part of Coordinate geometry. In higher study we draw graph say for the variations of some parameters, simplest the flow between parallel plates, say velocity and y distance. These are situation, where such curves intersect and at the point of intersecticn, we have equal values of velocity and y distance. The flow we know is divided into three parts i.e. laminar, transition, turbulent (1)



LAMINAR FLOW IS BETWEEN a a' c d AND THE SURFACE OF THE PLATE

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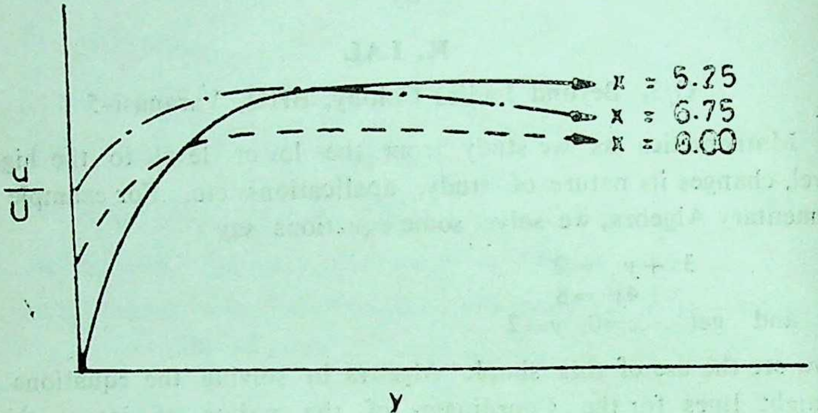
Diagram for boundary layers on a flat plate.



Mean velocity profiles in the transition region on a flat plate

Above graphs are given to show one example where we solve the problem by considering the point of intersection and important conclusions are drawn.

Another example, we may take of flow through the open channel which we come across every day. This area is of practical applications, several evamples, we find in Hydraulics and Fluid Mechanics (2)

(1) Irrigation practices along the Nile in Egypt and in the Valleys of the Tigris and Euphrates in Mesopotamia were known in 3000 B. C.

(2) Water supply, irrigation and flood protection techniques along the Indus river in India, and in China were known in 2000 BC.

(3) Original Suez Canal (Egypt)–a channel 64 km. long connecting the eastern branch of the Nile delta to the Bitter Lakes which were possibly open to the Red Sea is very old i. e. 1500 BC.

(4) The Romans constructed more than 560 km. of masonry apueducts to bring into Rome city in 312 BC–130 AD.

This shows that mathematics and its applications are quite old. We know the use of simple mathematics in such problems. We use certain formulae in such cases as such:

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(i) Chezy's equation (French Engineer 1775), given by

$$V = C.\,(RS)^{1/2}$$

where V = Velocity

C = Chezy's coefficient

R = hydroulic mean radius = A/P

A = area of cross-section of flow

P = wetted perimeter (wetted length of the chennel boundary)

S = Bed slope of the channel.

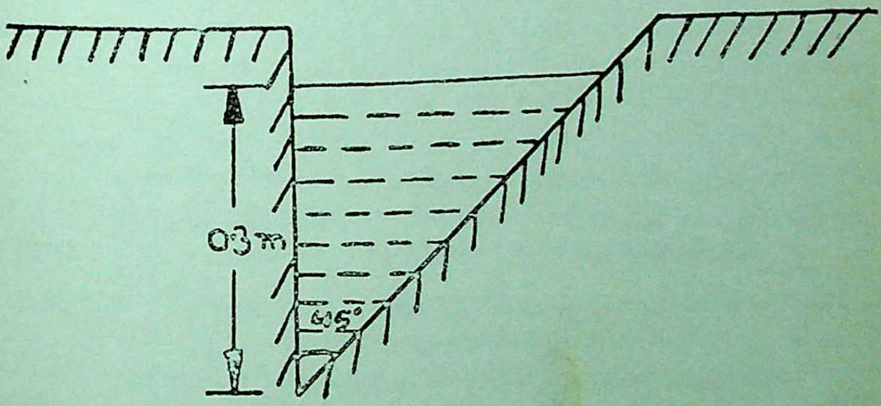
(ii) Manning formula (after Irish Engineers, 1891)

$$V = 1/n\; R^{2/3}\; S^{1/2}$$

where n = Manning's roughaess coeffecient.

Now we give one example of practical application which we see even in Haridwar the road side. (Ref2, *P*. 467).

A triangular road side gutter has one side vertical and the other side is at 45^0. It is laid at a slope of $\frac{1}{2}$ percent. Determine the storm flow when the water is 30 cm. deep. Assume $n = 0.028$



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$$Q = AV$$
$$= \left(\tfrac{1}{2} \times 0.3 \times 0.3\right) \frac{1}{0.028} \left[\frac{\tfrac{1}{2} \times 0.3 \times 0.3}{0.3 + ((0.3^2(0.3)^2)^{1/2}}\right]^{2/3} .$$
$$.(0.005)^{1/2}$$
$$= 0.018 \quad \text{cumec} = 18 \text{ lit per second (lpc)}$$

These are several examples like above one where we use simple geometry of high shool to calculate as in above figure, area of the cross-section, length of the side of a triangle etc. The august gathering here is actively engages in higher research in various branches of Mathematics and they all know the use of Mathematics in Technology, Medical Sciences, Social Sciences, Political Science, Jyotish Sciences etc.

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S. RAMANUJAN- A Historical Perspective

By

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Mr. Chairman and Honourable Delegates:

It is a pleasure to be here on the occassion of first Annual Conference of Vijnana Parishad of India and Symposium on Vedic Mathematics- Traditions and Applications.

In 1987 we celebrated the centenary celebrations of great son of India S. Ramanujan and I think it would be in best Vedic traditions (though not in the from of Shlokas and Mantras) to pay homage to him who in a way incarnated a few qualities of Vedic Rishis.

SRINIWAS AIYANGAR RAMANUJAN AIYANGAR was born on December 22, 1987 at Erode a suburb on the banks of river Kaveri. His father was a Munshi at a Cloth merchant shop. His mother was a pious lady and a great devotee of Goddess Namgiri. He regarded his father as Shiromani (best) of all males and his mother as incarnate of mother Parwati. Super criative activity in him was both intelect and intuition combined which he took to be a gift of Goddess. He passed his School leaving certificate examination in 1903 in first division. In 1909 he was married to one Janaki Devi. Now the worry of bread and butter made him to search for a suitable job as he could not fare well at F. A. examination. With the efforts of V. Ramaswami Aiyer (founder of Indian Mathematical Society in 1907), P. V. Sheau Aiyar and S. Narayan Aiyar he was able to get the post of a clerk in Madras Port Trust but to supplement his earnings he had to take to private tutions also.

Although, as mentioned above, he failed in F. A. examination he had acquired proficiency in Indeterminate forms in third standard, Loney's trigonometry in fourth standard but he was initiated into higher mathematics from a simple book "A Synopsis of Elementary Results in Pure and Applied Mathematics" by George Carr.

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January 26, 1913 was Makar Sankrati an auspicious day for Hindus. This day proved to be auspicious too for modern Indian Mathematics. On this very day on persuasion and inspirtaion from P. V. Sheshu Aiyer S. Ramanujan wrote a long letter to Prof. Hardy of Trinity College England describing among other things his work on divergent series and integrals. Before coming into contact with Prof. Hardy Ramanujan had not seen any standard book on Elliptic functions. This is evident from the fact he did not recognise many characteristic properties of elliptic function like addition theorem etc. Also he used his own notations.

As it would have it Indian Mathematical Society was founded in 1907 by V. Rama Swami Aiyar and his group consiting of R. P. Prajpe, M. T. Narayan Aiyangar among others. Again in 1909 Ashutosh Makerji was instrumental in shaping Calcutta Mathematical Society and in 1919 Ganesh Prasad founded Banaras Mathematical Society (Presently Bharata Ganita Parishad Located at Lucknow University at present). These societies started three journals the Journal of Indian Mathematical Society, Bulletin of Calcutta Mathematical Society, and Proceedings of Banaras Mathematical Society (at present "GANITA"). Thus the history of modern Indian Mathematics begins with last half of first decade of this century. It was within this period that Ramanujan's genious stated showing off. His first paper enititled "Some properties of Bernoulli's Numbers" was published in the Journal of Indian Mathematical Society in 1911. (Vol. III December No .), though one of his earlier papers (in the form of questions) had appeared in Vol. III February number. Two more appeared in 1912.

Coming back to biographical sketch, it is aignificant to note that on receipt of previously mentioned letter from Ramanujan Prof. Hardy tried hard to persuade him to come to Cambridge. He deputed Prof. E. H. Neville for the purpose. And on March 17, 1914 they proeeded to Cambridge. However prior arrangements had to be made for supplementing, as desired by Ramanujan, the poor pecuniary condition of his parents (in the form of Rs. 60/- p. m. out of his scholarship to be paid to his parents) and by instructing streamer agents of S. S. Nevasa for providing strictly vegetarian food through

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out the voyage.

During the five odd years of stay at Cambridge Ramanujan published 21 papers in different journals of Europe of which five were in Collaboration with Hardy.

As mentioned earlier before going to England he did not use the familiar Legendre $k, k' K, K', E E'$, etc. neither he used Jacobian constant q or the Weierstrassian g_2, g_3, e_1, e_2, e_3, etc. It was only in his papers "Modular equations and approximations" and " On certain arithmetical functions" he uses the modern notations. He was so modest in nature that while proving his basic identity he writes "The elementary proof of these formulae given in the preceeding sections seem to be of some interest in itself". He proceeds to give a very simple proof of the Discriminant identity of Jacobi. It may be noted that the proof of Ramanujan's differential equation is entirely algebraic in character as also his other developments. The most intricate part of Ramanujan's work is the modular equations given in various forms and the evaluation of the corresponding singular modulii.

On February 28, 1918 (again on an auspicious day-MAHASHIVA-RATRI to be particular) Ramanujan was elected a fellow of Royal Society - the highest honour then in scientific world. He was (contrary to popular belief) second Indian *F. R. S.*, the first being Ardeseer Cursetjee, who was elected in 1847. On October 13, 1918 (during Dussehara) Ramnujan was elected a fellow of Trinity College, the first Indian to receive this honour. During this period Madras University (from 1st April 1919 to be exact) granted an yearly allowance of Pounds250 for him which he gratefsully acknowledged and wished that out of the amount "Pounds60 an year (be paid to my parents and that the surplus be used for poor boys and orphans". This shows his parents and havenots.

He had a great affection for his wife too. According to his personal friend Prof. G. C. Chatterjee he felt morose and sad if he did not receive regular letters from his wife. Prof Neville recalls " The wonderful mathematician was a lovable man. " He was a good conversationalist and a better companion. He was quite simple and humility incarnate. If he was devoted to anyone he was devoted to last· In Trinity he

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was referred to as " Dear Jam " He took delight in discussing politics though he abhored indulging in the same for personal ends. According to Hardy he was a keen philosopher and an ardent politician of a pacifist and ultra radical type". But above all he was a great devotee of Goddess Mathematics in the true sense. Undaunted by poverty, depression of not getting success at F. A. examination, insistance on the part of his father to leave doing Mathematics he carried on his work on Magic Squares, Continued fractions, Hyper-geometric series, Theory of Numbers, Partition of Numbers, Elliptic functions, Fractional differentiation etc. He hated wars and refused to solve problems connected with war. So much so that he started abhoring applied Mathematics.

Any way being undernourished because of his orthodox nature and having food habits entirely unsuited to cold climate of England his health declined. He caught pulmonary tuberclosis. His worries for his parents and wife unnerved him and had a ruining effect on his health. So on Feb. 17, 1919 he had to leave England aboard S. S. Nagoya. He was to Chetput for convalseing. But as he prohesied to his wife that at Chetput every thing (including his death) would be "Chatput" he breathed his last on April 26, 1920, at the age of 32 years 4 months and 4 days only.

On January 20, 1920 from his sick bed Ramanujan wrote to Hardy in all humility " I am extremely sorry for not writing you a single letter uptill now. I have discovered a very interesting function which I call Mock-I function". He was perhaps mocking also at the mockery of Maya.

He has gone but his Mock Theta function and Roger Ramanujan identity remain and so do his Note Books As one of his biographers Sri Suresh Ram writes. "There can be no greater tribute or memorial to any genious than his work being devoutly taken up by those coming after him". Let us remember him by doing and practising mathematics,

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SOME EXAMPLES IN ALGEBRA

By

SUNDER LAL

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1 INTRODUCTION

Ganit is not only the science of *ganana,* but is also the study of *gana* (*smooh* or sets). That is why some people define mathematics as the study of sets and structures on sets. This is more aptly true about algebra where we study concepts like groups, rings and fields. These are all sets together with some structures on them. A structure is made up with the help of certain definitions and axioms. It is then given a proper shape with the help of theorems and results that are proved for these structures. In between, there lie examples. These examples are used to illustrate definitions and theorems. However, to demonstrate the limitations of a theorem, we need counterexamples. In the present talk we will discuss some new examples, and counterexamples in some algebraic structures like groups, rings and fields.

2. GROUPS :

We begin with $(Z, +)$, the additive group of integers, It is perhaps the simplest ezample of a group that one finds in an abstract algebra book. We may be little unorthodox and define 'sum' of a and b by translating $a + b$ by 1. In other works, we define an operation $\oplus$ on Z by setting

$$a \oplus b = a + b + 1 \text{ for all } a, b \in Z$$

It is quite easy now to verify the various group axioms. Here -1 is the identity (neutral) element and $b = a$-2 is the inverse of a in Z.

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Going into the verification of the group axioms one finds quite interestingly that here 1 plays no special a role. Any integer positive or negative would have worked. Thus if $n \in Z$, and if $a \oplus b$ is defined by setting

$$a \oplus b = a + b + n, \text{ for all } a, b \in Z$$

then $(Z, \oplus)$ is a group. Here the identity element is $-n$ and the inverse of a is $b = a - 2n$. By giving different values to n we may get as many examplэs as we like. Thus from the group $(Z, +)$, we get infinitely many examples of a group.

We may apply the technique used in the preceding paragraph to multiplicative groups also. Let, for example, $(Q_0,)$ denote the multiplicative group of non-zero rational numbers Let q be a fixed non-zero rational number, and led $\odot$ be detined by setting $a \odot b = abq$. One can easily verify that $(Q_0, \odot]$ is a group Here q^{-1} is the identity element, and $b = (aq^2)^{-1}$ is the inverse of a. Here again we get, gixen one example, infinitely many examples of a group.

In the example $(Z, \oplus)$ we need, for the verification of the group axioms, not only the fact that the operation $+$ satisfies these group axioms, but also the fact that it is a commutative operation. Similarly to verify that $(Q_0, \odot)$ is a group, we use the fact (and nothing more) that Q_0 is an *abeliau* group under multiplication of rational numbers. Using this observation we may generalize the previous examples further. Suppose $(G, +)$ *is an abelian group. Let g be a fixed element of G. If we define* $\oplus$ by *setting*

$$a \oplus b = a + b + g \text{ for } a, b \text{ in } G,$$

then $(G, \oplus)$ *is an abelian group*. The inverse $-g$ *of g in* $(G, +)$

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is the identity element in ($G, \oplus$). Also $-(a+2g)$, the inverse of $(a+2g)$ in $(G, +)$, is the inverse of a in $(G, \oplus)$.

Among the simplest examples of non-commutative operations are the subtraction of numbers and the division of numbers. However, none of the sets of numbers (integers, rationals, reals or complex) is a group with respect to any of these two operations. When we search for the examples of a non-abelian group, we find these in the set of (non-singular) matrices, in the set of permutations or in the set of transformations or mappings. This raises a very natural question : Do we have a non-abelian group of numbers ? The answer is yes, infact the examples are quite simple.

Consider our familiar set Z of integers. We define a new 'addition' $\oplus$ by setting

$a \oplus b = a + b$, if a is an even integer,

$= a - b$ if a is an odd integer.

It is obvious that Z is closed with respect to this new addition. In fact, it is only the associativity here which needs some detailed verification, the rest of the axioms are quite easy to verify. The integer zero is obviously the neutral or identity element. Also if a is an even integer then its inverse is $-a$, the negative of a, whereas if a is odd, then it is its own inverse. $(Z, \oplus)$ is obviously a non-abelian group. This group has many interesting properties. For example, it is an infinite group containing infinitely many elements of finite order (each odd integer is an element of order 2). Similarly. if H is a non-trivial subgroup of $(Z, \oplus)$, then H is either an infinite group or else contains only two elements. Incidently $(Z, \oplus)$ also happens to be a non-abelian group all (non-trivial) subgroups of which are abelian.

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If we define a new multiplication $\odot$ on Q_0, the set of non-zero rational numbers by setting $a \odot q = ab$ if a is positive, and $a \odot b = ab^{-1}$, if a is negative. then we can show that (Q_0, $\odot$) is a non-abelian group . Similarly, the set R_0 of non-zero real numbers is also a non-abelian group with respect to $\odot$.

The concept of identity or neutral element is very important in the study of an algebraic structure . When we introduce it for the first time to our students, we do not forget to emphasize that the identity element may be different from 1 (the usual multiplicative identity) even if the set under consideration consists of numbers and the operation is the multiplication . The most common examples of multiplicative groups are (Q_0, $\odot$), (R_0, $\odot$), (Co, $\odot$), ($Z(p) = \{1, 2, ..., p-1\}$, $\odot_p$) for a prime p, and the group $E(m) = \{x \in N : x < m, (x, m) = 1\}$, $\odot_m$) for < a positive integer m . Here $\odot_n$ stands for the operation 'multiplication modulo n'. In all these examples the identity element is 1. Some books discuss examples like ($\{1, 4, 7, 13\}$, $\odot_{15}$), where again the identity is 1. Some students, therefore, wonder whether there are examples of multiplicative groups of numbers where the identity is different from 1. Fortunately, examples as simple as ($\{2, 4, 8\}$, $\odot_{14}$) are sufficient to make life easy for him as the element 1 is altogether absent here and the identity element is 8. In this example, we may write the set as $\{2, 2^2, 2^3\}$, and 14 as $2+2^2+2^3$. One can prove that if p is a prime and n is any positive integer, then ($\{p, p^2, ..., p^n\}$. $\odot_x$) is a group, where $x = p + p^2 + ... + p^n$, The identity element of this group is p^n .

We mentioned earlier the group ($Z(p)$, $\odot_p$), where $Z(p) = \{1,2, ...p-1\}$, $\odot_p$ is the operation 'multiplication modulo p', and p is a prime . It is one of the first examples that one is exposed to in a course on abstract algebra. Another example is the well known group

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$E(n) = \{m \in N : m < n . (m, n) = 1\}$ with $\odot_n$ as the group operation. The order of this group is $\phi(n)$, where ϕ is the Euler's 'ϕ-function', and n is any fixed positive integer. A generalization of $Z(p)$ is the group $2Z(p)$, where the set $2Z(p) = \{2, 4, 6, \ldots, 2(p-1)\}$ and the operation is the multiplication modulo $2p$.

We obtained $(2Z(p), \odot_{2p})$ from $(Z(p), \odot_p)$ by multiplying elements of $Z(p)$ by 2 and also by replacing p of $\odot_p$ by $2p$. We may consider $(qZ(p), \odot_{qp})$, where q is any prime less than p. We can show that $qZ(p) = \{q, 2q, \ldots, q(p-1)\}$ is a group with respect to multiplication modulo qp.

Before we generalize the above example further, let us consider a generalization of $(E(n), \odot_n)$. As $(2Z(p), \odot_{2p})$ is a group one may guess that $(2E(n), \odot_{2n})$ is also a group for every n. This, however, is false. For if we take $n = 10$, then $E(10) = \{1, 3, 7, 9\}$ and therefore, $2E(10) = \{2, 6, 14, 18\}$. This set, however, is not closed with respect to multiplication modulo 20. Shall we now put some restriction on n? The n for which our guess fails is even. Why then not try with odd n? Let us consider $S = 2E(n) = \{2m : m \in N, m < n, (m, n) = 1\}$ for some positive odd integer n, and consider the operation 'multiplication modulo $2n$', which we denote by*. It can be proved that $(S, *)$ is a group.

The example that we are going to discuss now is a simultaneous generalization of the example of the preceding paragraph and the example preceding that. In the group $(2E(n), \odot_{2n})$ of the previous example, 2 is a prime and also $(2,n) = 1$. We may therefore, like to consider the set $qE(n)$, where q is a prime, and also relatively prime to n. Also in the example preceding the last one, we considered $q \leqslant (p)$, where both q and p were prime. Here also we may like to go from

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$q\,Z\,(p)$ to $qE\,(n)$ for the same reasons as those for which we moved from $Z\,(p)$ to $E\,(n)$. However, for some combination of q and n, $qE\,(n)$ may pose the same type of problem (viz. the closure property) as we faced with $2E\,(n)$, when n was even. The difficulty there was avoided by choosing n to be an odd integer, Here that alone may not work, For example, if $q=3$ and $n=9$. then $qE\,(n)$ is not closed for $\odot_{qu}$. The fact that n is odd can also be expressed (in relation to 2) by saying that it is relatively prime to 2. This suggests that we may try with $qE\,(n)$ where q is not only a-prime, but q and n are relatively prime too. Thus both the considerations lead us to the set $S=qE\,(n) = \{\, qm : m \lhd n, (m,n)=1 \,\}$ where q is a prime. $q < n$ and $(q, n)=1$. We can show that S is a group with 'multiplication modulo qn'.

As final generalization, we consider the set $m\,E\,(n)=\{mx\}: x<n, (x, n) = 1\,\}$, where $m < n$ and $(m\,n) = 1$. and the operation multi-pli cation modulo mn, It can be proved that ($m\,E\,(n)$, $\odot_{mn}$) is indeed a group.

3. **RINGS :**

We start with the following problem taken from the second edition of Herstein's *Topics in Algebra* (problem No. 11 on page 136) :

Let (R, +,) be a ring with unit element. Using its elements we define a ring (R, $\oplus_1$, $\otimes_1$) by defining.

$$a \oplus_1 b = a + b + 1, \text{ and } a \otimes_1 b = a + b + ab,$$

where $a, b \in R$ and where the addition and multiplication on the right hand side of these relations are those of (R, +,).

(a) Prove that (R, $\oplus_1$, $\otimes_1$) is a ring.

(b) What acts as the zero-element of (R, $\oplus_1$, $\otimes_1$) ?

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(c) What acts as the unit element of (R, $\oplus_1$, $\otimes_1$)?

(d) Prove that (R, $\oplus_1$, $\oplus_1$) is isomorphic to (R, $+$,) .

(In the matter of notations in the above problem we have departed from Herstein, but it is done with a purpose) .

The above mentioned problem provides a method to construct from a special ring (viz. a ring with unit element) another ring of the *same* type .

We apply the above methed to the ring (Z, $+$,) of all integers which is a ring with unit element, we get another ring of integers with unit element zero. But what about ($2Z$, $+$,), the ring of even integers ? It is a ring without the unit element . Is it possible to apply some modified form of the above method to ($2Z$, $+$,) to get another ring of even integers ? Since 1 is not an even integer, $\oplus_1$ defined above fails even in the closure property. We therfore, modify it and define

$$a \oplus_2 b = a + b + 2 \text{ for all } a, b \text{ in } (2Z, +,) .$$

It is easy to see that ($2Z$, $\oplus_2$) is an abelian group with -2 as the neutral element Thus -2 is the zero element of our proposed ring . If we retain $\otimes_1$ as such, we find that it does not distri-bute over $\oplus_2$. In fact

$$a \otimes_1 (b \oplus_2 c) = ab + ac + 2a + a + b + c + 2, \text{ where as}$$

$$(a \otimes_1 b) \oplus_2 (a \otimes_1 c) = ab + a + b + ac + a + c + 2.$$

Thus, for example, if $a = b = c - 2$. then $a \otimes_1 (b, \oplus_2 c)$ is 20 whore as the value of (a $\otimes_1 b$) $\oplus_2$ ($a \otimes_1 c$) is just 11. We, therefore, have to modify $\otimes_1$ too. We let

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$$a \otimes_2 b = a + b + ab/2 \text{ for all } a, b \text{ in } (2Z, +\ ,).$$

Since a and b are even integers, $2Z$ is closed for $\otimes_2$. The associativity of $\otimes_2$ and the two distributive laws $\otimes_2$ and $\oplus_2$ can be verified easily. Thus $(2Z, \oplus, \otimes_2)$ is a ring. Its zero element is -2 and the unit elemeut is 0. Note that this time we started with a ring without unit element, bnt what we get is a ring with unit elcment. The new ring is, theeefore, a richer ring. Obviously the two rings ase different property wise.

$(2Z, +,)$ is not the only ring of integers which is without a unit element. In fact for each integer n, $(nZ, +,)$ is a ring (consisting of all multiples of n) and it is without a unite element iff $|n| > 1$. The above method can be applied to all such rings (without a unit element) to get rings with unit element. In nZ we defiue $\oplus_n$ and $\otimes_n$ by setting, for all ab in nZ

$$a \oplus_n b = a + b + n, \text{ and } a \otimes_n b = a + b + ab/n.$$

It is easy tö see that $(nZ, \oplus_u \otimes_n)$ is a ring, in which $-n$ acts as the zero-element and the integer zero works as the unit element. Further, for $|n| > 1$, the rings $(nZ, \oplus_n, \otimes_n)$ and $(nZ, +,)$ are not isomorphic.

For $n = -1$, the ring $(nZ, \oplus_n, \otimes_n)$ is $(Z, \oplus_{-1}, \otimes_{-1})$ where

$$a \oplus_{-1} b = a + b - 1 \text{ and } a \otimes_{-1} b = a + b - ab.$$

However, $a+b-1$ and $a+b-ab$ are valid expressions in *any* ring with unit element. Therefore, if $(R, +, \otimes)$ is a ring with uuit element 1, then $(R. \oplus_{-1}, \otimes_{-1})$ is also a ring with unit element. Its zero element is 1 and the unit etement is 0, the zero element of $(R, +,)$.

New given a ring $(R, +,)$ with unit element, we can get two more rings $(R, \oplus_1, \otimes_1)$ and $(R, \oplus_{-1}, \otimes_{-1})$ both with unit elements,

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the unit elements in the two rings being the zero of $(R, +,)$. We want to generalize this method . Note the special natare of these 'generating' elements 1 and -1 . In a ring with a unit element, the two elements are invertible, in fact self-invertible . This suggests a possible generalization of the above construction as follows :

Let $(R, +,)$ be a ring with unit element. Suppose R contains a self invertible element g *which commutes with every element* of R. We define $\oplus_g$ and $\otimes_g$ by setting for all a, b in R

$$a \oplus_g b = a + b + g, \text{ and } a \oplus_g b = a + b + gab .$$

We can show that $(R, \oplus_g, \otimes_g)$ is a ring .

For a final ueneralization, let $(R, +,)$ be a ring with a unit element. Let g be an invertible element in R whose inverse commutes with every element of R. For a, b In R, we set

$$a \oplus_g b = a + b + g, \text{ and } a \otimes_g b = a + b + g^{-1} ab .$$

Here again it is easy to see thet $(R, \oplus_g, \otimes_g)$ is ring with nnit element .

If in the original problem $(R, +,)$ happens to be a commutatixe ring then $\otimes_1$ is also a commutative operation and henee $(R, \oplus_1, \otimes_1)$ is a commutative ring. The same is true about the ring $(R, \oplus_g, \otimes_g)$. We now ask a more specific question .

Is $(R, \oplus_g, \otimes_g)$ an integral domain if $(R, +,)$ is so ?

The same question can be asked for skew fields *i.e.*, division rings) and fields. Fortunately the answer to these questions is yes . We can show that if the ring $(R, +,)$ is a commutative ring (integral domain, skew field or a field), then so is $(R, \oplus_g, \otimes_g)$.

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As a final generalization we can show that if (R, $+$, .) is a field and if n is a positive integer. then (R, $\oplus_1^n$, $\otimes_1^n$), where

$$a \oplus_1^n b = a + b + n, \text{ and}$$

$$a \otimes_1^n b = na + nb + n(n-1) + ab,$$

is also a field. The zero-element of the field is $-n$, whereas $-n+1$ is tbe uuit element.

4. FINITE FIELDS

With finite fields we have more interesting situations. In addition to the fields obtained by using the methods discussed in the earlier section, we can generate here more fields of more interesting nature, We begin with $J_p = \{0, 1, 2, \ldots, p-1\}$. If p is a prime then (J_p, $\oplus_p$, $\otimes_p$) is a field (in what follows $\oplus_x$ and $\otimes_x$ will denote respectively the *addition modulo* x and the *multiplication modulo* x). First we show that certain composite numbers too can be used to generate fields. Consider, for example, $2J_p = \{0, 2, 4, \ldots, 2(p-1)\}$ for a prime $p \geqslant 3$. We know that ($2J_p$, $\oplus_{2p}$) is an abelian group and that the even integer $p+1$ is identity element of this group. Since the two operations $\oplus_{2p}$ and $\otimes_{2p}$ are addition and multiplication of numbers, the two distributive laws hold. Thus ($2J_p$, $\oplus_{2p}$ $\otimes_{2p}$) is a fietd with 0 as the zero element and $p+1$ as the unit element.

In the above generalization of (J_p, $\oplus_p$, $\otimes_p$) to ($2J_p$, $\oplus_{2p}$, $\otimes_{2p}$) one speciality of the number 2, that we use is that it is a prime, This suggests the possibility of (qJ_p, $\oplus_{qp}$, $\otimes_{qp}$) being a field where q is also a prime (distinct form p). This indeed is the case. It can be proved that it is a field.

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Similarly, if p be a prime, and if m be an integer relatively prime to p, then the system $(mJ_p, \oplus_{mp}, \otimes_{mp})$ is an integral domain and hence a field .

We close this account with rather unusual examples . In these examples the number zero is altogether absent and both the zero element and the unit element are greater than 1.

1. Let p and q be two primes (not necessarily distinct).

   Let $F = \{ p, p^2, \dots, p^q \}$, and let $x = p + p^2 + \dots + p^q$.

   For p^i, p^j in F, we set

   $$p^i \oplus p^j = p^r, \text{ where } p^r \equiv p^{i+j} \text{ (modulo } x \text{), and}$$

   $$p^i \otimes p^j = p^s, \text{ where } p^s \equiv p^{ij} \text{ (modulo } x \text{).}$$

We can show that $(F, \oplus, \otimes)$ is a field, that p^q is its zero element and that p is the unit element .

2. Let p and q be primes and let $F = \{ p^2, p^4, \dots, p^{2q} \}$.

   Let $x = p^2 + p^4 + \dots + p^{2q}$. If we set

   $$p^i \oplus p^j = p^r, \text{ where } p^r \equiv p^{i+j} \text{ (modulo } x \text{), and}$$

   $$p^i \otimes p^j = p^s, \text{ where } p^s \equiv p^{ij} \text{ (modulo } x \text{),}$$

then $(F_1 \oplus, \otimes)$ is a field with d^{2q} as the zero-element and p^2 as its unit element .

3. Let p, q and r be primes and $b \neq r$. Let $F = \{p^r, p^{qp}, \dots, p^{qr}\}$ and let $x = p^r + p^{2r} + \dots + p^{qr}$. If for p^i, $p^j F$ we set $\oplus$ and $\otimes$ as defined above, then $(F, \oplus, \otimes)$ is a field . Its unit element is p^r and the zero-element is p^{qr} ,

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4. Let p and q be primes and let m be a positive integer relatively prime to q Let $F = \{ p^m, p^{2m}, \ldots, p^{qm} \}$, and

let $x = p^m + p^{2m} + \ldots + p^{qm}$. Again we set $\oplus$ and $\otimes$ as in Example 1 above. Then $(F, \oplus, \otimes)$ is a field with p^m as the unit element and p^{qm} as the zero-element.

By giving different values to the integers p, q, r and m in the above examples, we can generate infinitely many (finite) fields with zero element different from the usual zero and the unit element diffeerent from the integer 1

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A BINOMIAL ANALOGUE OF THE CLASS OF ADDITION THEOREMS OF SRIVASTAVA, LAVOIE AND TREMBLAY AND ITS APPLICATIONS

By

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1. INTRODUCTION

Srivastava [5] presented certain interesting classes of generating functions in the forms :

$$\sum_{n=0}^{\infty} f_n^{\alpha+\lambda n}(x+ny)\frac{t^n}{n!}$$

and

$$\sum_{n=0}^{\infty} g_n^{(\alpha_1+\lambda_1 n,\ \ldots,\ \alpha_s+\lambda_s n)}(x_1+ny_1,\ \ldots,\ x_s+ny_r)\frac{t^n}{n!}$$

where

$$\left\{f_n^{\alpha}(x)\right\}_{n=0}^{\infty} \text{ and } \left\{g_n^{(\alpha_1,\ \ldots,\ \alpha_z)}(x_1,\ \ldots,\ x_s)\right\}_{n=0}^{\infty}$$

are general one and many-parameter sequences of functions. Further Srivastava, Lavoie and Tremblay [6] derived some general addition formulas for analogous sequences of functions and gave a number of interesting applications of the main results . Recently . Chandel and Sahgal [4] also gave a number of interesting applications and extensions of the addition theorems of Srivastava, Lavoie and Tremblay [6] .

In the present talk, a binomial analogue of the class of addition theorems of Srivastava, Lavoie and Tremblay [6] is discussed and a number of interesting applications of the main results are given .

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2. Theorem.

Let the functions $B(z)$ and $z^{-1}C(z)$ be analytic in the neighbourhood of the origin, and assume (for the sake of simplicity) that

(2.1) $B(0) = C'(0) = 1.$

Define the sequence of functions $\left\{ G_n^{(\alpha,\beta)}(x) \right\}_{n=0}^{\infty}$ by means of

$$(2.2) \quad [B(z)]^{\alpha} [1 - x\, C(z)]^{-\beta} = \sum_{n=0}^{\infty} G_n^{(\alpha,\beta)}(x) \frac{z^n}{k!},$$

where α, β and x are arbitrary complex numbers independent of z. Then for arbitrary parameters λ and μ independent of z,

$$(2.3) \quad G_n^{(\alpha+\lambda\gamma,\, \beta+\mu\gamma)}(x) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{n+\gamma}{k+\gamma}\right) G_k^{(\alpha-\lambda k,\, \beta-\mu k)}(x)$$

$$G_{n-k}^{(\lambda(k+\gamma),\, \mu(k+\gamma))}(x),$$

provided that Re $(\gamma) > 0$,

3. Applications of the Theorem

There are several applications of the theorem but for brevity, here we give only few examples.

Recently Chandel and Bhargava [1,2] have studied the polynomials which for $s=1$, are defined as

$$(1-z)^{-c}\left[1 - \frac{xz}{(1-z)^{r}}\right]^{-b} = \sum_{n=0}^{\infty} n!\ \Gamma_n^{(b,\,c)}(x, r, 1) \frac{z^n}{n!}.$$

Therefore, for $\alpha = c$, $\beta = b$, $B(z) = (1-z)^{-1}$ and $C(z) = z/(1-z)^{r}$, the above theorem gives

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$$(3.1)\quad \Gamma^{(b+\lambda\gamma,\ c+\mu\gamma)}(x,\ r,\ 1)$$

$$= \sum_{k=0}^{n} \left(\frac{n+\gamma}{k+\gamma}\right) \Gamma_{k}^{(b-\lambda k,\ c-\mu k)}(x, r,\ 1)\ \Gamma_{n-k}^{(\lambda\,(k+\gamma),\ \mu(k+\gamma))}(x,r,1).$$

Chandel and Bhargava [1,2] have also studied the polynomials associated to above polynomials whose generating function for $s=1$, is given by

$$(1+z)^{-c}\,[\,1-xz\,(1+z)^{r-1}\,]^{b} = \sum_{n=0}^{\infty} A_{n}^{(b,c)}(x,r,1)z^{n}.$$

Thus for $\alpha=c$, $\beta=-b$, $B(z)=(1+z)^{-1}$ and $C(z)=z\,(1+z)^{r-1}$, an appeal to above theorem gives

$$(3.2)\quad A_{n}^{(b+\gamma\lambda,\ c+\mu\gamma)}(x,\ r,\ 1)$$

$$= \sum_{k=0}^{n} \left(\frac{n+\gamma}{k+\gamma}\right) A_{k}^{(b\ \ \lambda k,\ c-\mu k)}(x,r,\ 1)\ A_{n-k}^{(\lambda\,(k+\gamma),\ \mu\,(k+\gamma))}(x,\ r.\ 1).$$

For $s=1$, the polynomlals of Chandel and Bhargava [3] are defined as

$$[\,C\text{-}mxz + yz^{m}\,]^{p} \left[1-\frac{r^{r}xz}{(C\text{-}mxz + yz^{m}\,)^{r}}\right]^{-q}$$

$$= \sum_{n=0}^{\infty} B_{n}^{(p,q)}(\,m,\ x,\ y,\ r,\ r.\ c\,)\ z^{n}\ .$$

Therefore, for $\alpha=p$, $\beta=q$, $B(z)=C\text{-}mxz+yz^{m}$ and

$$C\,(z) = \frac{r^{r}z}{C\text{-}mxz+yz^{m}}.$$

an appeal to the above theorem shows that

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$$(3.3)\quad B_n^{(p+\lambda\gamma,\, q+\mu\gamma)}(m, x, y, r, 1, C)$$

$$= \sum_{k=0}^{n} \binom{n+\gamma}{k+\gamma} B_k^{(p-\lambda k,\, q-\mu k)}(m, x, y, r, 1\, C)\, B_{n-k}^{(\lambda(k+\gamma),\, \mu(k+r))}(m, x, y, r, 1, C).$$

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ABSTRACTS OF THE PAPERS PRESENTED IN THE FIRST ANNUAL CONFERENCE OF VIJNANA PARISHAD OF INDIA HELD AT HARDWAR

(March 10-11, 1989)

A PHYSICIST'S WAY OF CHEKING THE CORRECTNESS OF MATHEMATICAL EQUATIONS

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Our message in this article is that a physical principle known as 'dimensional analysis' can be used to check the correctness of mathematical equations (i. e. equations from algebra calculus, geometry, matrices, etc.) and appreciate their beauty.

HYDRO—MAGNETIC STABILITY OE PARALLEL SHEAR FLOWS BETWEEN TWO POROUS PLATES

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In This Paper we have discussed the hydromagnetic stability of parallel shear flows between two porous plates. Growth rate of unstable modes has been obtained. Problem for non-oscillatory modes is also discussed and different results are obtained for different cases.

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A TWO PHASE MATHEMATICAL MODEL AND STATISTICAL ANALYSES FOR DRAG REDUCTION IN LAMINAR FLOW

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In this paper we have studied the effect of the presence of a very thin layer of slightly visco-elastic fluid very close to the wall analytically. The presence of this layer may be attributed either to the dissolution of the coated polymers on the wall or to the wall adherence phenomenon. It has been assumed that a slightly viscoelastic fluild is being continously and uniformly injected through the wall of the circular tube in which a purely viscous Newtonian fluid is flowing. The problem has been analysed in the two regions, one of purely viscous fluild and other of thin layer near the wall seperately alongwith suitable matching cond tions Finally, we have presented the numerical results and statistical analyses for drag reduction and total flux through a cross section for a particular pressure gradient with or without the presence of thin layer of polymer additives,

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THE FACTOR 10^{27} IN HINDU NUMERATION AND THE INDETERMINANCY PRINCIPLE

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The purpose of the present paper is to put forward the espeoial role of the facter 10^{27} in tenable Hindu Numeration on the basis of Sri Ram-Charitamanasa, styled as 'The Perfect of the Vedas', and to consider it the basis of the Heisenherg Indeterminancy(or Uncertainty) Principal in describing the laws of nature. The coincidence is especially important in the discussion of the tenabilities of the law of causality and the indeterminancy of infinity, and in the better understanding by contemplation of the processes involved, leaving more play for work of the imagination.

ON THE MIXED—BOUNDARY—VALUE PROBLEM IN THEORY OF ASYMMETRIC ELASTICITY

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The stress concentration problem in asymmetric elasticity around a circular hole in an infinite elastic medium has been analysed. In classical elasticity theory, the solution of a fundamental problem with mixed boundary conditions is available. However, no solution of such problems in asymmetric theory of elasticity is available. In

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this paper the fundamental problem with mixed boundary conditions which generally know as the third fundamental problem for an infinite plane with a circular hole has been solved, For this, tangential traction, normal displacemeat and rotation potential are prescribed at the boundary of the hole and tractions at infinity. Complex variable technique is applied to obtain the exdlcit general solution. As particular cases, few examples are discussed. Performing numerical calculation, a comparison of stress-distribution among asymmetric elasticity, centro-symmetric elasticity and classical elasticity has been done. It has been found that the result obtained in asymmetric case gives closer approximation to the classical case case than to the centro-symmetric case. Moreover, It has been found that the effects of stresses and couple-stresses are governed by Poisson's ratio and two parameters which are dependent on the material property of the asymmetric elastic medium.

HETEROGENEOUS POPULATION GROWTH MODEL

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The Solution of the modified Kendall's population growth model equation [1, 2] have been obtained by the outhor by taking the marriage rate as an quadratic function of time [3]. In the present paper we have considered a heterogeneous population growth with respect to the marraiage rate K when it is taken as the combination of a linear and exponential function of time. Hence we have

$$K=A_1\, t + A_2\, e^t + A_0$$

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where A_1, A_2 and A_0 are non - negative constants. The population growth model equations are given by

$$\frac{dM}{dt} + a_1 M = (a_3 + u_1)z - (A_1 t + A_2 e^t + A_0)$$

$$\frac{dF}{dt} + a_2 F = (a_4 + u_2)z - (A_1 t + A_2 e^t + A_0)$$

and

$$\frac{dZ}{dt} + \alpha Z = A_1 t + A_2 e^t + A_0,$$

where $\alpha = a_3 + u_4$ and M, F, Z are the number of unmarried males, unmarried females, and married couples respectively. a_1, a_2 are the death rates of unmarried males and unmarried females per unit time. Integrating the above equations and neglecting the second and higher degree terms, we get

$$Z = \frac{1}{\alpha}\left[A_1 t + \frac{\alpha A_2 e^{\alpha t}}{1+\alpha} + A_0\right] - \frac{A_1}{\alpha^2} + C\, e^{-\alpha t}$$

where C is the constant of integration given by

$$C = Z_0 - \frac{1}{\alpha}\left[A_0 - \frac{A_1}{\alpha} + \frac{\alpha A_2}{1+\alpha}\right],$$

where Z_0 being initial population of married couples We investigate the solutions for the number of numarried males and females and the total population

$$P = M + F + 2z.$$

The necessary and sufficient conditions have been obtained when the population of married couples, unmarried males and unmarried females are independent of time. The ratio of unmarried male

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population to the married male population is given by

$$\left(\frac{u_1 - a_4}{a_1} \right).$$

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MHD COUPLE STRESS FLUID FLOW PAST A STRETCHING SHEET

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In the present study, a steady two dimensional flow of couple stress fluid past a stretching sheet under the action of magnetic field perpendicular to sheet is considered. Similarity transformation yields governing equation into ordinary non-linear equation.. By using trial solution the problem is solved, and results are expressed in terms of dimensionless couple stress parameter K, and discussions have been made.

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SOME THEOREMS ON GENERATING FUNCTION

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So far a number of techniques are used to obtain generating functions. But no systematic attempt has been made to got generating functions through finite difference and shift operators. In this note an attempt has been made in this direction. We have given in all four theorems about generating functions. All the results are obtained with the help of the finite difference and shift operators. One of the theorems thus obtained is the extension of a result given earlier by Rainville These theorems as special cases includes a number of results scattered in the literature (only two or three cases are discussed for brevity).

The theorems proved are as follows :

THEOREM 1. If

$$e^t\, G(xt) = \sum_{n=0}^{\infty} f_n(x) t^n \quad , \qquad G(u) = \sum_{n=0}^{\infty} d_n\, u^n$$

Then

$$F(xt) = \sum_{n=0}^{\infty} \frac{((a))_n\, f_n(x)}{((b))_n}\, {}_AF_B \left[\begin{matrix} (a)+n; -t \\ (b)+n, \end{matrix} \right] t^n$$

where, $F(u) = \sum_{n=0}^{\infty} \frac{((a))_n\, d_n}{((b))_n} u_n$, and $((a))_n$ stands for

$$(a_1)_n \;,\dots\, ,\, (a_A)_n\, .$$

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THEOREM 2. Let the polynomials $f_n(x)$ be difined by (2.1).

Then

$$(1-t)^{-a} H(xt/(1-t), y(1-t)) =$$

$$\sum_{n=0}^{\infty} (a)_n f_n(x) \; {}_{B+2}F_D \left[\begin{matrix} -m, a+n, (b); y \\ (d); \end{matrix} \right] t^n \quad (2.3)$$

where, $$H(u,v) = \sum_{n=0}^{\infty} (a)_n d_n \; {}_{B+2}F_D \left[\begin{matrix} -m, a+n, (b); v \\ (d); \end{matrix} \right] u^n.$$

THEOREM 3. If the polynomials $f_n(x)$ be difined by (2 1). Then it follows that

$$\sum_{n=0}^{\infty} f_n(x) \; {}_{A+1}F_B \left[\begin{matrix} -n, (a); y \\ (b); \end{matrix} \right] t^n$$

$$= \sum_{n,r=0}^{\infty} \frac{d_n x^n}{r!} \; {}_{A+1}F_B \left[\begin{matrix} -n-r, (a); y \\ (b); \end{matrix} \right] t^{n+r} \quad (2.4)$$

and

THEOREM 4. If $\{ y_n(x) \}$ be a set of polynomials defined by

$$A(t) \exp\{-xt/(1-t)\} = \sum_{n=0}^{\infty} y_n(x) t^n,$$

$$\text{where } A(u) = \sum_{n=0}^{\infty} e_n u^n$$

Then

$$(1+t)^{-a} f(t/(1+t)) = \sum_{n=0}^{\infty} \frac{(a)_n \, y_n(x)}{(b)_n}$$

$$F^{(2)} \left[\begin{matrix} a+n:-; \; b+n; xt, -t \\ b+n:-;-; \end{matrix} \right] t^n, \quad (2.5)$$

where $$f(u) = \sum_{n=0}^{\infty} \frac{(a)_n \, e_n \, u^n}{(b)_n}$$

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VISCO-ELASTIC FLOW THROUGH POROUS MEDIUM BOUNDED BY AN OSCILLATING RIGID PLATE IN A SLIP REGIME

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In the present paper, we deal with the study of unsteady flow of Maxwellian fluid through porous medium near an oscillating plate with slip boundary conditions. Closed form solution for velocity profile has been obtained and the influence of time-relaxation parameter on it is discussed graphically. Numerically, we discuss the veriation of velocity with rare - faction and permeability of porous medium. Also the expression for skin-friction has been derived and related results are discussed numerically.

m-DIMENSIONAL GENERATING RELATIONS FOR $_{B}A_{N}$ $\{(x_m),y\}$

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In our earlier work (To appear in Bull. Cal. Math. Soc.), we have introduced a generalized polynomial system $_{B}A_{N}$ $\{(x_m),y\}$ of m variables with the help of generating relation including the generalized hypergeometric function $_{p}F_{q}$ and the Lauricella function F_{A}. This paper enunciates a linear and a bilateral generating relation for this polynomial system. Finally several multilinear and multilateral generating relations have been deduced as special cases of our results.

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FLOW IN A ROTATING POROUS PIPE

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Navier-Stokes equations of viscous incompressible flow, in a rotating frame of reference, given by Squire (1956), have been generalised to account for the porosity of the wall, with which the flow is in contact. Assuming uniform rate of suction of the liquid at the wall an approximate solution, valid for small values of both angular velocity and suction velocity, has been determined for flow undet constant pressure gradient through a circular pipa.

Results of Barua (1954) have veen recovered, as a particular case, when there is no porosity. The stream lines in the central plane and the projection of the stream lines on the cross-section of the pipe are compared with those in the non-porous case. An expression for the flux has been, also obtained.

LATTICE DYNAMICS OF TRANSITION METALS

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A theoretical model is used for calculating the lattice dynamical properties of hcp transition metals.

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DECOMPOSITION OF CURVATURE TENSOR IN R−⊕ RECURRENT FINSLER SPACE WITH NON-SYMMETRIC CONNECTION

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The purpose of present paper is to obtain some relations under the decomposition of curvature tensor R^h_{ijk} in R-⊕ recurrent Finsler space F^* equipped with non-symmetric connection parameter.

We consider n-dimensional Finsler space F_n^* with non-symmetric connection parameter $\Gamma^i_{jk}(x,\dot{x})$, that is based on non-symmetric fundamental metric tensor $g_{ij}(x,\dot{x}) \neq g_{ji}(x,\dot{x})$. We write Γ^i_{jk} as below:

$\Gamma^i_{jk} = M^i_{jk} + \frac{1}{2} N^i_{jk}$, where M^i_{jk} and N^i_{jk} denote symmetric and skew-symmetric parts of Γ^i_{jk}.

The covariant derivatives are defined in two ways:

$$T^{\overset{i}{+}}_{\underset{j}{+}} / k = \partial_k T^i_j - (\dot{\partial}_m T^i_j)\, \Gamma^m_{pk}\, \dot{x}^p + T^m_j \Gamma^i_{mk} - T^i_m \Gamma^m_{jk} \quad \ldots (1)$$

and

$$T^{\overset{i}{-}}_{\underset{j}{-}} / k = \partial_k T^i_j - (\dot{\partial}_m T^i_j)\, \tilde{\Gamma}^m_{pk}\, \dot{x}^p + T^m_j \tilde{\Gamma}^i_{mk} - T^i_m \tilde{\Gamma}^m_{jk} \quad (2)$$

Some theorems and results have been established. We mention here some results which have been established

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(I) $$(a_{\underset{++}{lm}} - a_{\underset{++}{ml}}) \mid n = \lambda_n (a_{lm} - a_{ml}) - \lambda_n I_{lm} + I_{\underset{++}{tm}} \mid n$$

$$+ \delta n \lambda r \, N^r{}_{ml} + a_{rn} \, N^r{}_{ml} + \lambda_r \, N^r{}_{\underset{+\;++}{ml}} \Big| \, n \, , \tag{3}$$

where
$$I_{lm} = \delta_l \lambda_m - \delta_m \lambda_l \tag{4}$$

(II) $$a_{[\underset{++}{lm/n}]} + a_{[lm\lambda_n]} + a_{[ln\delta_m]} + \delta_{[\underset{+}{m/n\lambda_l}]} + P_{lr} \, N^r{}_{nm}$$

$$+ P_{mr} \, N^r{}_{ln} + P_{nr} \, N^r{}_{ml} = 0 \, , \tag{5}$$

where
$$P_{lr} = a_{lr} + \lambda_l \delta_r \, . \tag{6}$$

(III) $$\phi_{kh} / [lmn] + (P_{lr} \, N^r{}_{nm} + P_{mr} \, N^r{}_{ln} + P_{nr} \, N^r{}_{ml}) \, \phi_{kh} = 0. \tag{7}$$

INEQUALITIES OF CERTAIN CLASSES OF ORTHOGONAL POLYNOMIALS

By

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Inequalities of general pollaczek polynomials, which generalize the random walk, ultraspherical and Coulomb potential polynomials are studied, Inequalities of basic Pollaczek polynomials are then introduced which include in its fold as special cases, contineous q-ultrasberical Laguerre and Hermite polynomials and sieved orthogonal polynomials of first and second kind. Also included in the discussions are the inequalities of birth and death process polynomials and of random walk polynomials associated with birth and death process, which give rise to inequality for Carlitz - Karlin-Mc Gregor (CKM) polynomials in the particular case.

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ACCELERATION COVARIANCE IN MHD TURBULENT DUSTY FLOW

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In this paper we have considered MHD turbulent dusty flow of an incompressible, viscous fluid which is nearly isotropic and spatially homogeneous. The expression for acceleration covariance has been derived and solution has been obtained in terms of difining scalars.

AN APPLICATION OF GENERALISED MEHLER FOCK TRANSFORMATION

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Vidisha, M. P.

The Testing Space Rp(I) for Generalized Mehler Fock Transformation and various properties of dual space $R'p(I)$ of $Rp(I)$ have been given in a series of papers by authors. This paper contains the application of transformation in evaluating the function by solving a boundary value problem in theory of elasticity. An infinitely long aniostropic, axisymmetric cylinder has been taken as a body and the functton satisfies the Laplace equation in cylindrical coordinates. The first part of paper deals with the proof of a lemma while second part deals with the formation and solution of the problem.

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VIBRATION ANALYSIS OF A NON-UNIFORM CIRCULAR PLATE SUBJECTED TO ONE DIMENSIONAL STEADY STATE TEMPERATURE DISTRIBUTION

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and

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The effect of temperature on the vibration of solid bodies is significant especially in mechanical constructions where certain parts of the body have to operate under elevated temperatures. The interest in this direction has highly increased because of rapid developments in space technology, high speed atmospheric flight and in nuclear energy applications. The present paper aims at discussing the effect of linearly transient temperature fields on the natural frequencies of a circular plate. The frequencies, defiection and moments corresponding to the first two modes of vibration are computed for clamped and simply-supported edge conditions by using Frobenius method.

GRAPHICS : FROM CANVAS TO COMPUTER

(A mathematical overview)

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Technical Teachers Training Institute, Bhopal

In this paper a brief of processes involved in transfer of any graphic primitive from real-world to computer display is given. A mathematical overview of curves and surfaces in geometric design is also presented.

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A NOTE ON MHD FLOW OF AN INCOMPRESSIBLE CONDUCTING FLUID OVER AN INCLINED NON-CONDUCTING IMPERVIOUS BOUNDARY

By

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Govt. Postgraduate College, Rishikesh

Some features of the flow of a viscous incompressible fluid over an inclined non-conducting impervious boundary with a magnetic field transverse to the flow, have been studied in this paper. It is shown that this MHD flow is self- superposable and an irrotational flow on which it is superposable is determined. some observations have been made about the vorticity end stream function of the flow by using the property of superposability and self-superposability.

SOME INTEGRALS INVOLVING THE PRODUCT OF THE DOUBLE HYPERGEOMETRIC FUNCTION AND MULTIVARIABLE H-FUNCTION

By

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Using the recent result of Srivastava and Singh [Rend. Circ. Mat. Palermo II, 32 (1983), 157-187], six general formulas (four definite integral and two countour integrals) involving the products of double hypergeometric functions and multivariable H-function have been established.

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SYSTEMS ANALYSIS APPROACH TO TOWN PLANNING 'A CASE STUDY OF NATIONAL CAPITAL REGION'

By
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Roorkee University, Roorkee

B. B. Garg and Kushum Shanker
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In the present paper, we have considered the feasibility of using systems analysis approach to determine the optimal assignment of excess population of Delhi to different towns of the National Capital Region. In the problem it is desired to maximixe the efficiency of the system at the minimum possible cost.

ON A TWO VERIABLE ANALGUE OF HERMITE PLYNOMIALS

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In the present paper authors have defined a two - variable analoge of Hermite functions, $H_{k,n}^{(\nu)}(x,y)$ by generating function :

$$e^{-(t^2+T^2)}(1+2xt+2yT)^{\nu}=\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{H_{k,n}^{(\nu)}(x,y)}{k!\,n!}t^{k}T^{n}$$

This two variable analogue happens to be associated with two variable Kampe - de - Feriet's hyper geometric function. Explicit forms, recurrence relations, expansions and relations in terms of other well known classical polynomials have been obtained. Finally n-variable analogue of Hermite polynomials have also been defined.

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INVERSION OF AN INTEGRAL INVOLVING GENERALISED HERMITE FUNCTION

By

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The purpose of the present paper is to obtain the inversion of ihe Integral

$$g(x) = \int_0^\infty k(x/y)\, f(y) \frac{dy}{y},\ x > 0$$

where

$$k(x) = x^a \exp(-px^r)\ H_n^{(\alpha)}(x, a, p).$$

A NOTE ON NÖRLUND MEAN OF ORTHOGONAL SERIES

By

Dhanesh Patel, S. R. Agrawal and C. M. Patel
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M. S. University, Baroda

In the present paper we prove the following theorem for Nörlund means using the (L) condition.

Thenrem : Let (P_n) be a nonnegative monotone sequence of real numbers such that $P_n \to \infty$ as $n \to \infty$ and $np_n = O(P_n)$. If an increasing sequence of natural numbers (ν_n) satisfies the condition (L), than we have under the condition (1.2), the relation

$$S_{\nu_n}(x) - t_{\nu_n}(x) = o_x(1),$$

almost everywhere.

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GENERALIZATION OF FERMAT'S LAST PROBLEM

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Fermat's Last Problem:

There exist no natural numbers $a, b, c,$ for $n>2$ such that $a^n + b^n = c^n$ where n is any natural number.

Generalized Fermat's Problem :

There will exis no natural numbers $a_1, a_2, \ldots, a_m, b$ such that

$$a_1^n + a_2^n + a_3^n + \cdots + a_m^n = b^n \qquad \ldots \text{(A)}$$

for $n>m$ and where m, n are natural numbers, while for $n<m$, natural numbers $a_1, a_2, \ldots, a_m, b$ exist which satisfy equation (A). Few illustrations are given below:

Illustrations:

1. For $m=n=3$ we have natural numbers 3, 4, 5 and 6 such that

$$3^3 + 4^3 + 5^3 = 6^3$$

2. For $m=5$ and $n=4$ we have natural numbers 2, 2, 3, 4, 4 and 5 such that

$$2^4 + 2^4 + 3^4 + 4^4 + 4^4 = 5^4$$

3. For $m=3$ and $n=2$ we have natural numbers 1, 2, 2 and 3 such that

$$1^2 + 2^2 + 2^2 = 3^2$$

4. For $m=4$ and $n=3$ we have natural numbers 1, 1, 5, 6 and 7 such that

$$1^3 + 1^3 + 5^3 + 6^3 = 7^3$$

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ON GENERATING RELATIONS OF POLYNOMIAL SET $\left\{ U_n^*(x,y) \right\}$

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In this paper a polynomial set $\left\{ U_n^*(x,y) \right\}$ has been considered by means of the generating relation

$$\sum_{n=0}^{\infty} U^{*\, b;\lambda;\lambda_1;\lambda_2;V;l_1;l_2;(c_r);(\alpha_g);(\beta_h);(a_p,A_p)}_{n;m;m_1;m_2;m_3;m_4;(d_s);(y_\phi);(v_\epsilon);(b_{q+1},B_{q+1})}(x,y)\, t^n$$

$$= P\, F^{r:g;h}_{s:\phi;\epsilon}\left[\begin{matrix}(c_r):(\alpha_g);(\beta_h);\\(d_s):(y_\phi);(v_\epsilon);\end{matrix}\ (bx^{\lambda_1} t^{\lambda_2}, (\lambda y^{-m} t^{m_1})\right] \times$$

$$H^{l_1,l_2}_{p,q+1}\left[-V x^{m_2} y^{m_3} t^{m_4} \,\middle|\, \begin{matrix}\{a_p, A_p\}\\(b_1,1), \{b_{q+1}, B_{q+1}\}\end{matrix}\right];$$

where $E^{r:g;h}_{s:\phi;\epsilon}\left[\begin{matrix}(c_r):(\alpha_g);(\beta_h);\\(d_s):(y_\phi);(v_\epsilon);\end{matrix}\ (bx^{\lambda_1} t^{\lambda_2}), \lambda y^{-m} t^{m_1})\right]$

are Appel functions of two variables represented in most modified notation due to Burchnall and Chaundy,

$$H^{l_1,l_2}_{p,q+1}\left[-Vx^{m_2} y^{m_3} t^{m_4} \,\middle|\, \begin{matrix}\{a_p, A_p\}\\(b_1,1), \{b_{q+1}, B_{q+1}\}\end{matrix}\right]$$

are Fox's H-function and

$$P = \frac{\prod_{j=l_2+1}^{p} \Gamma(a_j - A_j b_1) \prod_{j=l_1+1}^{q+1} \Gamma(1-b_j+B_j b_1)}{\prod_{j=1}^{l_2} \Gamma(1-a_j+A_j b_1) \prod_{j=2}^{l_1} \Gamma(b_j - B_j b_1)} \times (-Vx^{m_2} y^{m_3} t^{m_4})^{-b_1}$$

A number of generating relations for several interesting classes of polynomials have been obtained from the main result. Some useful transformations and series identities have been used to find some useful results. Direct applications of Srivastava and Carlitz have reduced the theorems to interesting special cases. It has been observed how readily these new generating functions can be extended to help the advancement in the field of special functions.

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SOME INTEGRAL REPRESENTTAIONS OF A GENERALIZED POLYNOMIAL SET $\left\{ M_n^* (x,y) \right\}$

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The present paper deals with integral representations of the polynomial set $\left\{ M_n^* (x,y) \right\}$. We have obtained finite single, infinite single and finite double integral representations for $\left\{ M_n^* (x,y) \right\}$ in terms of theorems and corollaries. Some special interesting cases have also been derived.

A GENERAL COINCIDENCE THEOREM

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A coincidence theorem generalizing Goebel's coincidence theorem, Jungck contraction principle, Matkowski's fixed point theorem, Fisher's generalization of Jungck theorem and coincidence theorems of Kominek and Singh-Gairola-Mehndiratta is established for four systems of transformations on a product of n arbitrary sets with values in metric spaces

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FIXED POINTS OF MULTIVALUED MAPPING IN PROBABILISTIC METRIC SPACE

S. L. Singh and **B. D. Pant**
G. K. University, **Govt. P. G. College**
Hardwar-249494 **Kotdwar (Garhwal)**

Let $C(X)$ denote the set of all nonempty subsets of a Menger space (X, F, t) and let $P, Q: X \to X$ and $S, T: X \to C(X)$. In this paper we prove common fixed point theorems for P, Q, S and T. These results are generalizations of some well known coincidence and fixed point theorems for single-valued and multivalued mappings in metric and probabilistic metric spaces.

AN EXTENSION OF ANCIENT INDIAN CUBE-ROOT METHOD

S. L. Singh and Ramesh Chand
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Hardwar-249 404

In this paper an attempt is made to extend Bhāskarāchārya's method of obtaining cube-root of a perfect cubic number to 5th root of any integer. The method indeed may be applied to compute n^{th} root of any positive integer.

A FIXED POINT THEOREM FOR A SEQUENCE OF MAPPINGS ON 2-METRIC SPACES

S. L. Singh and Vijayendra Kumar
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In this paper a fixed point theorem on a 2-metric space is presented

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SOME COMMON FIXED POINT THEOREMS IN 2-METRIC SHACES

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In this paper following result is established for a 2-metric space (X, d) and two self-mappings S and T :

If there exists real numbers p and q such that

$p \in (0, 1)$ and $p > 1 + q$ and

$$[d(x, Sx, a) + d(y, Ty, a)]\; b(x, Ty, a)$$

$$\leqslant q \max \{[d(x, y, a)]^2, [d(x, Sx, a)]^2, [d(y, Ty, a)]^2, (1/p)([d(x, Ty, a)]^2 + [d(y, Sx, a)]^2\}$$

where $x, y, a \in X$. Then S and T have unique common fixed point.

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VARIOUS FIXED POINT THEOREMS

By

Rekha Mehndiratta*

Research Scholar, G. K. University, Haridwar

Fixed point theory has been a topic of great interest among researchers. This theory has been of a great help to bridge the gulf between "Pure Mathematics and Applied Mathematics". Fixed point theorem given by Banach, Brouwer, Schauder, Glicksberg and Browder have important applications in Pnysics. Economics, Engineering and in some other Mathematical Sciences. these theorems along with their applications have been started here.

Definition (Fixed point) :- Let $T: X \rightarrow X$ where X is a metric space. If $x_0 \epsilon X$ be such that $Tx_0 = x_0$, than x_0 is called a fixed point of the operator T.

1. Banach's Contraction Mapping Theorem .

Let (x, d) be a complete metric space and T be an operator, mapping x into itself such that

$$d(Tx, Ty) \leqslant a\, d(x, y) \text{ for all } x, y \in X$$

where $0 < a < 1$.

Then there exists a unique fixed point of the operator T in X.

Banach's fixed point theorem has been applied to find the unique solution of linear algebraic equations, differential equations, integral equations and as well as to implicit function theorem.

2. Brouwer's Theorem :

Every continuous mapping of a closed bounded convex set in R^n into itself has a fixed point.

Brouwer's theorem has been applied to the theory of positive matrices which are important in economics, probability theory and in other fields.

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3. Schauder's Theorems.

(i) (Schauder's First Theorem) :

If S is a convex compact subset of a normed space, every continuous mapping of S into itself has a fixed point.

(ii) (Schauder's Second Theorem) :

If S is a convex closed subset of a normed space and R a relatively compact subset of S, then every continuous mapping of S into R has a fixed point.

Schauder's theorems have been applied to Swirling-flow problem, to a problem concerning nonlinear oscillations of a pendulum and in Quantum and Statistical Physics.

4. Fan-Glicksberg Theorem.

Let C be a non-empty compact convex subset of a normed linear space X. For $x \in C$, let $F(x)$ denote a non-empty closed convex subset of C. Assume the following property (called upper semi continuity of F):

If $x \in C$ and V is an open subset of C containing $F(x)$, then there is an open subset U of C containing x such that for all $x \in U$, $F(x) \subset V$. Then there exists $a \in C$ such that $a \in F(a)$.

Set valued maps often arise in the control theory for various systems in engineering and economics, in the theory of differential games etc. Fixed point theorems, such as the one above are useful in these situations.

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5. Browder's Theorem.

Let X be a uniformly convex Banach space and S, *a* non-empty closed bounded and convex subset of X. If Z is a family of non-expansive maps from S to S such that $FG = GF$ for all $F, G \in Z$ then Z has a common fixed point in S.

This has been proved to be of remarkable value. Browder himself has applied it to show the existence of periodic solutions of a very general class of non-linear equations of evolution in infinite dimensinal spaces.

A COMMON FIXED POINT THEOREM OF COMMUTING MAPPINGS IN SAKS SPACE

By

R. S. Chandel and Ashok Ganguly
Department of Mathematics
C. S. I. T. S., Indore - 452001, M. P., India

Common fixed point theorem for four mappings is proved in a Saks space.

In this paper, motivated by the work of Cho and Singh [Kobe J. Math. 3 (1986), 1-6] on Saks space, we prove a fixed point theorem for four mappings on a Saks space. Our theorem extends the result of Xie Ping Ding [Math. Sem. Notes, 11 (1983), 301-305] on the fixed point theorem for cummuting mappings on the line of Jungck [Amer. Math. Monthly, 03 (1976), 261-263.

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ON APPROXIMATE SOLUTIONS OF MULTIVALUED MAPPING IN ORDERED BANACH SPACES

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Magadh University
Bodh Gaya

Department of Mathematics
Maharaja College, Arrah
(Magadh University)

Let X and Y be ordered Banach spaces, $A : X \rightarrow Y$ be a linear mapping and $N : X \rightarrow Y$ a nonlinear mapping of some type. To solve problems in partial and ordinary differential equations, these problems may be described in terms of an operator equation

$$Ax + Nx = z \ (x \in X, z \in Y) \qquad ...(1)$$

It is now only required to solve the operator eqn. (1).

In This paper we have given some results on approximate solvability of multivalued mappings in ordered Banach spaces. The existence of non zero solution of eqn. (1) be proved. Theorem (1) is improved version of P. S. Milojevic. We have also given the more generalised versions of theorem (1) for the purpose of appropriate applications.

NORMAL STRUCTURE AND FIXED POINT THEOREMS IN CONVEX METRIC SPACE

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In this paper we have proved some fixed point theorems in convex metric space using the action normal structure. Banach space fells in the category of convex metric space. Hence here we have generalized the result of Kannan [Proc. Math. Soc.38 (1973), 111-118] and Kirk , Proc. Amer. Math. Soc Symp. Pure Math. 18 (1970), 162-168.

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स्वागत-भाषण

प्रो० एच० एन० सिलाकारी

प्राचार्य, सम्राट अशोक अभियांत्रिकीय संस्थान विदिशा (म०प्र०)

आज के उद्घाटन समारोह के सम्माननीय मुख्य अतिथि, मध्य प्रदेश के जन-शक्ति नियोजन मंत्री माननीय श्री महेश प्रसाद शुक्ल, समारोह के अध्यक्ष, विदिशा के लोकसभा सदस्य और इस संस्था की संस्थापक सोसायटी के सचिव माननीय श्री राघव जी भाई, भारतीय विज्ञान परिषद के पदाधिकारीगण, सभी उपस्थित विद्वज्जन एवम् सम्माननीय अतिथिगण !

आज भारतीय विज्ञान परिषद के इस द्वितीय वार्षिक अधिवेषन के शुभ अवसर पर विदिशा की इस प्राचीन भूमि पर मैं आपका हार्दिक स्वागत करता हूं। विदिशा नगरी प्राचीन काल में व्यापार, उद्योग, सुख, समृद्धि, कला और संस्कृति का एक विख्यात केन्द्र रही है और इस भूमि को वाल्मीकि पतंजलि, कालिदास, बालभट्ट, शूद्रक और श्री हर्ष जैसे विश्व विश्रुत दिग्गजों की कर्म भूमि होने का भी सौभाग्य प्राप्त हुआ है। विदिशा में वे तीक्ष्ण धार वाली तलवारें भी बनती थीं जिनके बल पर मौर्य नर-पतियों ने यवन आक्रामकों का मद मर्दन किया था और विदेशों में बौद्ध धर्म का शांति-सन्देश लेकर जाने वाले महेन्द्र और संघ मित्रा भी विदिशा की ही एक श्रेष्ठ-कन्या के पुत्र और पुत्री थे। परन्तु आखिरी गुणन फल में विदिशा के नागरिक एक शांति-प्रिय सभ्यता और संस्कृति के विश्वासी थे। विदिशा नगरी भारतीय पुरातत्व की गणना में अपने विशाल और उत्कृष्ट स्थापत्य और कला के नमूने के आधार पर एक अद्वितीय स्थान रखती है।

भारतीय विज्ञान परिषद के इस द्वितीय वार्षिक अधिवेशन और उसके साथ आयोजित संगोष्ठी में अनेकानेक महत्वपूर्ण विषयों पर अधिकारी विद्वानों के भाषण आज और कल होंगे। परन्तु चूंकि गणित और अन्य विज्ञानों के क्षेत्र में कार्यरत विद्वान इतनी बड़ी संख्या में यहाँ उपस्थित हैं, मैं आप सबसे अपना कुछ समय भारतीय विज्ञान की कुछ मूलभूत समस्यायों की चर्चा में भी देने का अनुरोध करूंगा जिनके कारण भारतीय विज्ञान को आज जहाँ होना चाहिये था वहां से काफी दूर है और कदाचित सही दिशा में भी नहीं है।

जब हम इस बात पर विचार करते हैं कि ९ अंकों और एक शून्य वाली वह क्रांतिकारी अंक पद्धति जिसके कारण ही विज्ञान ने बाद में तीव्र गति से प्रगति की, एक अनजान भारतीय की तीक्ष्ण बुद्धि का चमत्कार था और यह देखते हैं कि अर्वाचीन

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काल में अनेकानेक भारतीय गणितज्ञों और वैज्ञानिकों ने प्रथम कोटि के अनुसंधान किये हैं तो यह स्पष्ट हो जाता है कि हमारी विज्ञान की समस्याओं का हल निकालना और पुनः उस विश्व के वैज्ञानिक संसार में शीर्षस्थ स्थान पर पहुंचना जहां कि वह ३००—४०० वर्ष पहले तक भी था, किसी आकाश-फल को तोड़ लाने का स्वप्न नहीं है ।

मेरी अल्प बुद्धि में, भारतीय विज्ञान के सामने आज जो प्रश्न रखे जा सकते हैं, वे इस प्रकार हैं :

१. क्या भारतीय विज्ञान भारतीय है? इससे मेरा तात्पर्य किसी प्रकार के पुनरुत्थान वादी विचार से नहीं है । अपनी सशक्त परम्पराओं से जुड़ा होना पुनरुत्थान वाद नहीं है ।

२. क्या भारतीय विज्ञान आज जिस प्रशासनिक ढांचे में चल रहा है वह वैज्ञानिक प्रगति की आवश्यकतायें पूरी कर सकता है : अर्थात क्या नौकर शाही का जो शिकंजा हमारे वैज्ञानिक संस्थानों पर भी हावी है, वह सही व्यवस्था करेगा ? प्रो. जे बी. एस हाल्डेन ने अपने जीवन के उत्तरार्ध में भारत में आकर कुछ कार्य करना चाहा था । परन्तु जब उनका पाला हमारी नौकरशाही से पड़ा तो वह हताश होकर कुछ ही समय में भारत छोड़ कर चले गये ।

३ भारतीय विज्ञान में क्या उत्कृष्टता की प्राप्ति एक सर्वमान्य लक्ष्य बन सकी है ? (गुलाम मानसिकता बुद्धिजीवियों की मानसिक कायरता, सरकारी नौकरी का आतंक)

४. क्या भारतीय वैज्ञानिक भारतीय समाज की आर्थिक और सामाजिक समस्याओं को सुलझाने के प्रति प्रतिबद्ध है और उसके लिये सही दिशा पकड़ सका है ? उपयुक्त तकनीकी के प्रति हमारे वैज्ञानिकों में वितृष्णा है । राजनीतिक और आर्थिक निहित स्वार्थों द्वारा निर्धारित दिशा में वैज्ञानिक विकास और उसका सीमांकन काफी कुछ सुनियोजित ढग से किया जाता है जिससे हमारी वैज्ञानिक प्रगति एक सीमा से आगे नहीं जा पाती ।

५. भारतीय वैज्ञानिकों में कुंठा और कार्य संतुष्टि की भारी कमी का कारण क्या है ? कई वैज्ञानिकों की आत्म-हत्यायें और भारी मात्रा में प्रतिभा पलायन हमारे वैज्ञानिक संस्थानों के कुछ कटु सत्य हैं ।

६. क्या हमारे वैज्ञानिकों का दृष्टिकोण वास्तव में वैज्ञानिक है ? नेहरू ने कहा था कि हमारे वैज्ञानिक अपने सामान्य जीवन में अक्सर वैज्ञानिक दृष्टिकोण नहीं अपनाते । मैं समझता हूं कि वे अपने वैज्ञानिक कार्य में भी वैज्ञानिक दृष्टिकोण

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नहीं अपना पाते। वे unique solutions के प्रति आग्रह रखते हैं जिसके कुछ घातक परिणाम पर्यावरण और ऊर्जा की समस्याओं के रुप में हमारे सामने हैं। उदाहरण के लिये जिन्हें गैर पारंपरिक ऊर्जा श्रोत कहा जाता है उनमें से अधिकांश कई वर्ष पूर्व ईंधन पर आधारित ऊर्जा साधनों का ही हर जगह प्रयोग करने की अन्धी दौड़ में छोड़ दिये गये थे और आज ईंधन के विश्व व्यापी संकट के कारण हम उन्हें पुनः अपना रहे हैं। यह उपयुक्त तकनीकी के प्रति वितृष्णा और unique solutions के प्रति आग्रह का उदाहरण है। अपने विकल्पों को संकुचित करना वैज्ञानिक दृष्टिकोंण नहीं है।

यह कुछ प्रमुख प्रश्न हैं जिन पर विचार किये बिना हमारा वैज्ञानिक समाज अपने बौद्धिक, सामाजिक और राष्ट्रीय तकाजों को पूरा नहीं कर सकता।

इतना ही कहकर मैं पुनः आपका इस दो-दिवसीय अधिवेशन में स्वागत करता हूं और आशा करता हूं कि यह अधिवेशन हमारे वैज्ञानिक सोच-विचार पर अपनी कुछ छाप छोड़ेगा।

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AN INTRODUCTION OF "VIJNANA PARISHAD OF INDIA"

(Society for Applications of Mathematics)

By

R. C. Singh Chandel

Secretary, Vijnana Parishad of India, D. V. Postgraduate College
Orai-285001, U. P., India

Honourable Chairman Shri Raghavji, M. P., Chief Guest Shri M. P. Shukla, State Man Power Planning Minister, Professor M. K. Jain Vice-Chancellor, A. P. S. University, Rewa, Principal Shilakariji, Local Secretary Professor R. D. Agrawal, Delegates and Guests:

I am personally grateful to this assembly which has given me an oportunity to speak and introduce with the "VIJNANA PARISHAD OF INDIA", which is a society dedicated to Applications of Mathematics in all fields of Science and Technology and as such it has a crucial role to play because it is only the society in India explicitely devoted to this goal.

In 1970, Dr. B. B. Lal the Ex-Principal, D. V. Postgraduate College, Orai-285001, U. P. desired of his science teachers to start a publication of a Scientific Research Journal from the College, containing a treasure of quotable references for the help and guidance of research scholars and worth preservable by the erudite Libraries. Responding to his inspiration. Dr. R. C. Singh Chandel of Mathematics Department came forward to fulfil his dream and to facilitate the research activities and encourage the young researchers by taking of the publication of Mathematical Research Journal "JÑĀNĀBHA" ज्ञानाभ (Ray of Knowledge) published by "VIJNANA PARISHAD of D. V.

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Postgraduate College, Crai. It was started in 1970–71, with generous co-operation of various experts as active members of its Editorial Board, renowned Professor J. N. Kapur as its Chief Advisor Dr. B. B. Lal as Patron and R. C. Singh Chandel as Editor.

As soon as the Journal "JÑĀNĀBHA" saw the light of the day in 1971 it attracted the attention of the scholars of the World and crossed the boundary of the Country in no time. As it could not be kept limited to the College, the "VIJNANA PARISHAD" of D. V. Postgraduate College, Orai adopted slightly modified name "VIJNANA PARISHAD OF INDIA" in place of "VIJNANA PARISHAD" indicating an extension of its field to other countries, and besides having an independent status of its own free form the control of an institution. Here we can not forget the good name of Professor H. M. Srivastava (University of Victoria, Canada) who in 1972 was invited to work as a Foreign Secretary of the PARISHAD and who gladly agreed to work as one of the Editors. Some foreign experts of different fields of Mathematics and other Sciences were also included in the Editorial Board.

Since the begining of 1972 "VIJNANA PARISHAD OF INDIA" has Reciprocity Agreement with "AMERICAN MATHEMATICAL SOCIETY". The Parishad is one of the following five Indian Societies which have mutual "RECIPROCITY AGREEMENT" with "AMERICAN MATHEMATICAL SOCIETY": 1. Allahabad Mathematical Society 2. Calcutta Mathematical Society 3. Indian Mathematical Society 4. Ramanujam Mathematical Society, Mysore and 5. Vijnana Parishad of India.

The Mathematical content af the Journal "JÑĀNĀBHA" of "VIJNANA PARISPAD OF INDIA" is indexed in the American Biweekly " CURRENT MATHEMATICAL PUBLICATIONS " and is reviewed among others, by Mathematical Reviews (U. S. A.),

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Zentralblatt fur Mathematik (W. Germany) and Applied Mechanics Reviews (U. S. A.).

Upto 1985, the Parishad was an unregistered body and its aim was only to publish regularly the Research Journal "JÑĀNĀBHA". On 24th August, 1986 under the Presidentship of Professor J. N. Kapur a meeting was held at I. I. T. Kanpur and it was decided to get it registered. Consequently it was got registered on 4th July 1988 with D. V. Postgraduate College, Orai-285001, U. P. as its registered office having the following aims and activities:

AIMS OF THE PARISHAD

To work for the development of Mathematical Sciences in India. To encourage the interaction of Mathematics with Engineering, Industrial, Medical, Social, Physical, Biological and Computer Sciences. and specially

1. To work for the popularisation of Mathematics among students and public.
2. To work for introduction of the more "Applications of Mathematics" in curricula at all levels.
3. To work for greater interaction of Mathematics and users of Mathematics for the solutions of those national problems which require the use of Mathematics.
4. To intensify training and research in Industrial and Applied Mathematics.

ACTIVITIES

1. To publish Research Jornals and Monographs in Mathematical Sciences.
2. To publish Journals for popularising Mathematical Sciences and their teaching techniques.
3. To hold Conferences, Symposia Exhibitions and Fairs for the development of Mathematical Sciences.

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4. To establish Libraries for Research Scholars.
5. To encourage Mathematics Education.
6. To publish advance books.
7. To encourage the interdiciplinary activities.
8. To establish Local Chapters of the Parishad in different Universities,
9. To award prizes to the talented students.
10. To award prizes and honours to the teachers and researchers of India for their unique work of eminence.
11. To take other steps considered necessary to promote the aims of the Parishad.

At present Parishad has 75 Life Members including 25 Foreign members. Parishad gets 18 Journals in Exchange of JÑĀNĀBHA including 8 Foreign Journals,

First Annual Conference of the Parishad was held at Hardwar under the auspices of Gurukul Kangri University, Hardwar on March 10-11, 1989, partially supported by DRDO, New Delhi. As a part of the Conference, we also organised a National Symposium on "VEDIC MATHEMATICS: APPLICATIONS AND TRADITIONS". Since 1989, we have also started the publication of another Journal entitled "PROCEEDINGS OF VIJNANA PARISHAD OF INDIA". It publishes the Proceedings of the Annual Conferences of the Parishad.

Now here we are holding the Second Annual Conference of "VIJNANA PARISHAD OF INDIA", 1990 under the auspices of S. A. Techonlogical Institute, Vidisha, partially sponsored by CSIR, New Delhi, Council of Science and Technology, M. P., Bhopal, Barkatullah University, Bhopal, Britania Industries, Vidisha and Shiv Shakti Paper Mills, Vidisha. As a part of the Conference we are also organising a National Symposium on "APPLICATIONS OF MATHEMATICS IN MODERN SCIENCE AND TECHNOLOY

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This is quite appropriate since this Institute has many scholars of Science and Technology and as such this Symposium should provide an unique opportunity for a meaningful dialogue with technologists. For this we are very much thankful to Shri K. S. Dhillon, Vice-Chancellor Barkatullah University, Bhopal, Professor H. N. Silakari, Principal of thisInsitute and Professor R. D. Agrawal, Local Secretary. It is only due to their keen interest and encouragement that we could get together here at a very short notice.

It could not be out of place to mention here that since the very inception of the first publication of the Joural JÑĀNĀBHA or in other words since last 20 years we are carrying on our mission by our own resources and have received no financial aid form the Government or from any other agency. Further it is a matter of great satisfaction and appreciation of the Parishad that the Journal is being published regularly with poor financial resources and only scanty help from voracious readers, researchers and authors. Every one here can realize the difficulties which our Parishad has been facing for the publication of the Journal and fulfilling the aims of the Parishad. No organization or Association can run without monetory funds. Our Mathematical progress may too hemper for want of necessary funds. As such the Parishad has to look the perpetual resources from which we may continue to fulfil the aims of the Parishad. In case we happen stand care free in the financial domain for regular publication to the Journals and execution of other activities aimed at by the Parishad, we can carve out an important place for it in the Mathematical World, and the torch of our mission may give light to other countries too. Whatever financial hard-ship we may have to suffer, we shall never retreat. We are bound to go ahead. We shall continue our march to the goal we have decided, even in the circumstances contrary to our progress.

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If we use mathematics meaningfully, we can save hundreds of crores of Rupees for the country. There are many challenges for the mathematicians in the modern World. I am sure that the members of the Parishad will accept these challenges of placing India in the world Map of Industrial and Applied Mathematics. The Parishad seeks full co-operation of all of you in this task. I hope, we shall have very meaningful and exiting discussions in the Conference and shall be able to answer the questions raised by the Principal of the Institute, in his welome sheech, and the conference will give us a goal for which we shall work in the next year and in the years ahead.

I thank you once again brothers and sisters assembeled here from different parts of the country and welcome them, and also seak their full co-operation and good wishes for the success of the conference.

Jai Ganita, Jai Vijnana Parishad and Jai Hind

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VIJNANA PARISHAD OF INDIA

Income and Expenditure Account for the Year Ended 31st March, 1990

Income	Amount Rs.	Expenditure	Amount Rs.
Opening Balance	4182.70	Publication of Vol. 18	10885.00
By Subscription, Publication Charges and a Grant for Rs.5000.00 from DRDO, New Delhi	27326.65	Advance for Publication of Vol. 19	10000.00
		Postage etc.	1833·00
		Total	22718.00
		Closing Balance	8791.35
	31509·35		31509.35

(Sd.) J. N. Kapur
President

(Sd.) Tilak Singh
Society Auditor

(Sd.) R. C. Singh Chandel
Secretary–Treasurer

(Sd.) M. M. Lal & Company, C. A., Kanpur

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VIJNANA PARISHAD OF INDIA
SECOND ANNUAL CONFERENCE

S. A. Technological Institute, Vidisha, 1990

(BARKTULLAH UNIVERSITY, BHOPAL)

PRESIDENTIAL ADDRESS

B

Professor J. N. Kapur, *Ph. D.*; *F. N. A. Sc.*; *F. A. Sc.*; *F. N. A.*

INTRODUCTION

At the outset, I must express my sincere regrets at not being able to be present at the conference due to my absence in Canada, where I came to fufil an earlier obligation to work as a Visiting Professor at the University of Waterloo for a period of three months. However I send my best wishes for the success of the conference and the symposium on "Applications of Mathematics in Modern Science and Technology.

THE CONFERENCES I HAVE ATTENDED

After having come here I have attended two conferences-one of Operations Research Society of America at Las Vegas, U. S. A. and the other of Operations Research Society of Canada at Ottawa. The first conference had about 3000 delegates and worked at every time in 23 parallel sessions for at least 21 hours, so that there was about 500 hours of serious academic work in less than three days. In each session, four paers of 20 minutes duration each were presented. At least 2000 persons presented their papers and the time for each paper had been fixed about four months in advance. The second conference was smaller. In it about 200 persons presented their papers and there were seven parallel sessions at any time. I gave invited talks in both these conferences and in the second conference, I chaired a session on 'Entropy'.

Comparatively our conferences are much smaller and few have parallel sessions.

These conferences dealt with only one aspect of mathematical sciences viz. operations research. In our conference, we deal with all branches of pure and applied mathematics and if we want to work on the same

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scale as the developed nations, we should have at least one hundred sessions at any given time.

I am saying this to highlight our position in the world of science. These academic conferences do represent, in same sonse, the scale of academic activities in a country. We should have a target of increasing these activities by more than fifty times in the next two decades.

INCREASING FUNDS FOR RESEARCH

Increasing academic activities requires increase of resources, both human and financial. In our country, funds for scientific research have increased tremendously, since independence, but funds for mathematical sciences research have remained almost static. One reason is that we have failed to convince the public and the government of the tremendous importance of mathematics for national development. The other reason is the myth that mathematical research does not require much funds, due to which mathematicians themselves do not ask for large funds. However mathematics research requires fellowships, scholarships, salaries, library books and journals, computer time, building etc. in the same way as other sciences do; only we do not require equipments. Our research budget could therefore be fifty percent of theirs, it should certainly be not 1%of theirs.

As such the following steps are urgently needed:

(i) Every one of us should 'sell' mathematics to the public and the government. We should 'popularise' mathematics among the students and the masses. We should take all the steps to attract bright students to mathematics.

(ii) We should do intense lobbying for increased funds for mathematical researah with UGC, CSIR, DST., NBHM, with central and state governments and with industrialists and philanthropists.

(iii) We should increase our interaction with users and take greater interest in mathematical solution of national problems of science and technology.

(iv) We should introduce courses in industrial and applied mathematics and train future mathematicians in applications and computers in a big way.

(v) We should make better use of available funds. It is a fact that for various reasons, at least one-third of the funds sanctioned by the UGC and other agencies for mathematical sciences are not utilized.

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(vi) Mathematicians should submit more proposals in frontline areas of national interest to funding agencies.

APPLY AND COMPETE FOR RESEARCH GRANTS

In Western countries, the prestige of a university faculty menber is measured by the amount of research grants he can get even more that than it is measured by the number and quality of papers he has published. Moreover it is not easy to know where and how many papers a person has published. Under these conditions the amount of the research grant becomes an important index of a person's research ability. Funds for typing of papers, photo-copying travel for attending conferences, even for having research students usually come from research grants. As such, who do not have research grants are seriously handicapped in their research work and academicians do research of good quality to win research grants. For experimental scientists, it is even more essential because they can not sometimes have been an essential equipment unless they get a grant for it. In our country there is a greater competition among experimental scientists for research grants than there is among theortical scientists and among all scientists, theoritical or experimental, the number of mathematicians applying for research grants or even for research scholarships and fellowships is the smallest. This situation has to change soon if mathematical science research has to flourish in India.

The Parishad would therefore like to appeal to all its members and infact to all mathematicians to do research of high quality, apply for grants and improve their research contributions more and more. Fvery one should be ambitious for his research and should have very clear-cut goals for this work.

MY CURRENT AREAS OF RESEARCH

Over the last thirty-five years I have worked in a variety of fields viz. Internal Ballistics of Guns and Rockets, Fluid Dynamics, Magneto Hydro Dynamics, Operations Research, Population Dynamics, Mathematical Bioeconomics, Biomechanics, Social Decision Systems, Mathematical Economics, Flexible Manufacturing Systems, Maximum Entropy Models, Pattern Recognition and so on,

Currently I am interested in two main fields, both of them rather unorthodox viz. in Mathematical Models of Innovation Diffusion and Generalised Maximum Entropy Principles and their Applications and

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I would like to share with the members some of the exciting problems on which I have worked and results in these two fields who I have obtained. Both these fields belong to Applications of Mathematics to Modern Scince and Technology' on which weore having a symposium.

MATHEMATICAL MODELS OF INNOVATION DIFFUSION

In the present world, innovation spreads very rapidly. There may be innovation in products, in technologies and even in ideas.

New technologies substitute the old and are in course of time are substituted by newer tehnologies. The models which explain how innovations substitute and diffuse are very important for manufacturers marketing, specialists and government agencies.

During the last thirty years about a dozen mathematical models of innovation diffusion have been proposed by a number of workers. Each person proposed some model, took some real-life data and showed that his model explained the data very well.

Some of the models were obtained by considering external influences like those of radio, T. V. and mass media publicity and internal influences due to mouth-to-mouth and interpersonal communication of information detween the group those who are using the innovation and those who have yet to adopt it. Some other models were proposed on the analogy of epidemics because there is a great deal of resemblance between spread of epidemics and spread of innovation. Some models were obtained by considering that growth of innovation may be similar to growth of population. Still others were obtained by considering the nature of the diffusian process. In general when an innovation is introduced, the number of those who adopt it increases first at an increasing rate, reaches after some times its maximum rate of increase and after that number of innovations still increases, but at a decreasing rate. This if $f(t)$ is the proprtion of the potential adopters, which has adopted a new innovation till time t, then $f(t)$ first increases at an increasing rate so that f—t curve is convex and later it increases at a decreasing rate so that finally f-t curve is concave. For some value f^* of f, the function $f(t)$ changes from convexity to concavity and we get a point of inflexion (Figure 1).

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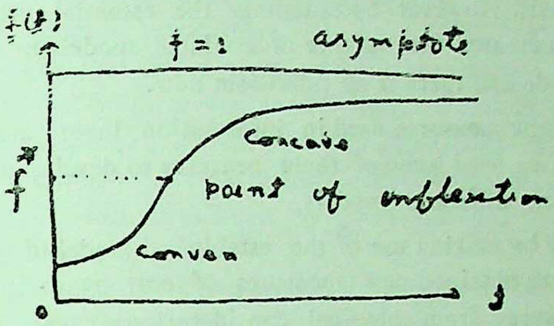

FIGURE 1

There is a point of inflexion at f^* when the function changes form convexity to concavity. Finally the entire market is saturated by new innovation and so $f(t) \to 1$ as $t \to \infty$ and $f=1$ is an asymptotic to this f—t curve.

As such, models with S-shaped curves are very important. If the mode is given by

$$\frac{df}{dt} = \phi(f),$$

then $\phi'(f) > 0$ as f goes from 0 to f^* and $\phi'(f) < 0$ when f goes from f^* to 1 and $\phi'(f) = 0$ when $f = f^*$ so that $\phi(f)$ is an increasing-decreasing function as shown in Fig. 2.

As a special case, $\phi(f)$ can also be a concave function of f.

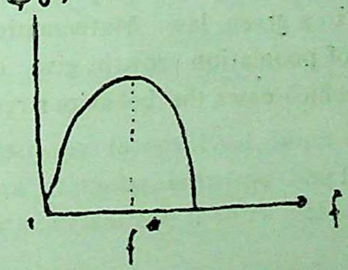

FIGURE 2

THE ROLE OF THE MATHEMATICIAN IN INNOVATION DIFFUSION MODELS

(i) As noted earlier, about a dozen innovation diffusion models were discovered over a period of three decades more or less on an

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ad-hoc basis. However by examning the essential properties of $\phi(f)$, we increased the number of available models by an order of magnitude and there is no adhocism now.

(ii) Some entropy measures used in information theory are concave functions. We used some of these measures to develop another set of inovation diffusion models.

(iii) Conversely by making use of the established model of innovation diffusion, we obtained new measures of entropy so that we got these measures from physical considerations rather than from purely axiomatic considerations.

(iv) 'Competition' between species has been studied extensively in mathematical ecology. We extended its results to competition detween technologies and products. The mathematical structure is the same whether there is competition between species or between technologies.

(v) If $N_1(t)$, $N_2(t)$ grow exponentially independently of each other then $N_1(t)/[(N_1(t)+N_2(t)]$ can grow in accordance with the logistic law. Thus we showed that the logistic behaviour noted in many cases may not be due to substitution, but may be due to both technologies growing exponentially.

(vi) In most models the total numbe of those who can adopt a new technology is assumed as fixed. Dynamic Innovation Diffusion Models were developed by postulating an external popula ion growing according to a grven law. Mathematically we discussed as to which laws of pooulation growth give rise to closed-from solutions and in which cases the behaviour remains logistic.

(vii) By considering a multi level model consisting of uniformed persons, informed and potential adopters and actual adopters we get dynamic models without postulating any external population growth law.

(viii) By solving a system of simultaneous differential equations, we considered models when the first technology is substituted by the second, second by the third, third by the fourth and so on.

(ix) There is a delay in communication of information about a new technology and its actual adoption. We developed models in

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terms of delay-differential and integro-differential equations to take into account these delay.

(x) We showed that under certain conditions, we can transform a given innovation diffusion model into another by transforming either the dependent or the independent variable. We used these transformations to discuss diffusion over both space and time as well as diffusion controlled by price changes or government subsidies.

(xi) Most of the models studied so far are deterministic. Mathematically we have been able to study some stochastic innovation diffusion models.

(xii) Most of the earlier models involved parameters. These were resetricted to certain ranges by those who proposed these because of the ways they derived these. We extended the ranges of these parameters significantly by considering the requirement that f^* has to lie between 0 and 1. We also discussed mathematically the variation of the time t^* to point of inflexion for different models.

The above list is not exhaustive. However it shows that if users can go themselves one mile, they can go ten miles with the help of mathematics. Mathematics is a powerful tool which allows us to exploit analogies meaningfully, which allows us to generalise, which allows ns to get deeper insight into the structure of these models and which shows relationships between models, not appearent otherwise. That is why all usere have a deep respect for mathematics. However only those mathematicians can be helpful, who are prepared to take deep interest in the user's field. If they do so, they may find, as we found, a large number of mathematically challenging problems in every field of application.

GENERALISED MAXIMUM ENTROPY PRINCIPLES AND THEIR APPLICATIONS

Newton gave his first basic law of mechanics in the form "Everydody continues in its state of rest or of uniform motion in a straight line unless it is acted on by an external force."

We have a corresponding law in probability theory: "Every probability distribution is a uniform distribution unless its moments are different from those of the uniform distribution".

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However while in the case of mechanics, if we konw the force vector acting in a body, we can always find the motion uniquely, in probabilty theory, if we know one or many moments, there may be more than one probability distribution cosistent, with the given moments. Which one should we choose ? The Principle of Maximum Entropy states that we should choose that probability distribution which is uncertain i. e. which maximizes the 'uncertainty' or 'entropy' of the probability distribution subject to the given constraints.

Just as in mechanics, where for given forces, we find the motion and for given motion, we find the forces, here also we have the inverse problem of finding the constraint which will make a given probability distribution as maximum entropy distribution.

We were the first to solve this inverse problem and discuss the existence and uniqueness of the solution. However we went a step further. We even allowed the measure of entropy to be flexible. Earlier a measure given by Shannon was invariably used, though some other measures were also given. We posed the problem of finding the measure of entropy by maximizing which subject to given moment constraints we can get a given probability distribution as maximum entropy distribution and in this way we generated measure of 'generalised entropies'.

Newton's law may also be stated in the form "The motion of everybody will continue to as near its state of rest of uniform motion in a straight line as possible subjeet tothe forces acting upon it". The basic law of probability theory can be stated in the form "A probability distribution will be as near the uniform distribution as possible subject to all probability constraints that are imposed on it". If the probabiity distribution prior to the imposition of constraints is Q, then the law is modified to state that "the probability distribution P after the imposition of the moment constraint C will be as near as possible to the prior distribution Q subject to the constraint C.

To implement this new law, we need a measure of 'distance' between the a posterior probability distance P and the a prior probability distance Q. One such measure was given by Kullback and Leibler in 1951. Other were given by Renyi, Havrda–Charvat and others. We also give some measures.

This gave rise to the Principle of Minimum Cross Entropy accoring to which "Given a priori probability distribution Q, a set of moment constraints C, a measure of probability distance (a cross-entropy or directed divergence) $D(P: Q)$, choose the a posterior probability P so that $D(P: Q)$ is minimum".

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This enabled us to give our generalised principle of minimum cross-entropy according to which "Given a probability distribution P and any two of the following (i) a priori probability distribution Q (ii) moment constraints C (iii) measure of directed divergence $D(P: Q)$, we should choose the third so that $D(P: Q)$ is minimum."

In case Q is the uniform distribution, this reduces to Generalised Principle of maximum entropy.

Thus in one sweep, we were able to increase tremendously the scope of applications of the classical principles of maximum and minimum crossentropy.

Thus consider the problem of estimating the proportions $p_1, p_2 \cdots\cdots p_n$ of persons being in n colonies when the constraint is on the travel budget to the Central Business District. The Classical maximum entropy principle give a very neat solution. However if there are further restrictions that the proportion in the ith colony should lie between a_i and b_i then Shannon's measure of entropy and the classical principle fail. We have to find an alternative measure of entropy. The generalised maximum entropy principle helps us in finding this measure.

We first ask "What is the most natural probability distribution wihch arises when the constraints are $a_i \leqslant p_i \leqslant b_i$ and $\sum_{i=1}^{n} p_i = 1$?" We call this distribution as the generalised uniform distribution. We next ask "what is the measure of entropy by maximizing which subject to $\sum_{i=1}^{n} p_i = 1$, we get this generalised uniform distribution as the maximum entropy distribution ?"

We thus generate a number of new measures of entropy (uncertainty). Similarly we generate a number of new measures of Directed divergence (distance) when there inequality constraints on probabilities. Thus a whole new class of problems becomes amenable to easy solution.

REFERENCES

The maximum entropy and minimum cross-entropy principles have had tremendous applications in statistics, economics, urban and regional planning, pattern recognition, transportation, operations research, thermodynamics,

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computerised, tomography, time-series analysis, computer science, statistical mechanics, chemical reactions marketing, finance, portfolio analysis and so on. The basic theory of these principles and their applications are described in my book "Maximum Entropy Models in Science and Engineering" published this year by Wiley Eastern in India and John Wiley in New York. It gives a panaromic view of applications of mathematics in a variety of fields of physical, social, biological and engineering sciences through one basic principle and one can realize the tremendous scope of applications of mathematics through this book.

The generalised maximum entropy principle and its applications have been described in a monograph by myself and Prof. H. K. Kesavan published by Sandford Educational Press, Universiry of Waterloo, Ontario, Canada.

A third book 'Entropy, Optimization Principles and their Applications is under publication. It will enunciate about twenty principles of the same power and strengh as classical principle of maximum entropy.

The major book on Innovation Diffusion Models at present is by Mahajan and Peterson and is published Sage Publishers of U.S A. I am planning to write book on the subject.

For understanding the tremendous charm and fascination of mathematical sciences, one can consult 8 volumes of my book on 'Fascinating World of Mathematical Sciences' published by Mathematical Science Trust Society C-766, New Friends Colony New Delhi. Another, good book in 3 volumes 'Mathematics, People and Problems published by Wadsworth of U.S.A.

The U.G.C. journal 'Mathematical Education' published by Macmillan India and edited by me along with other friends gives articles of interest to students and teachers of mathematics. Bulletin Mathematical Association of India published by Mathematical Sciences Trust Society also gives articles of the same type.

CONCLUDING REMARKS

I hope and trust that the conference will be a great success, that the delegates will go back with great enthusiasm about high quality teaching and first rate research, about popularisation of mathematics, about creating a general awareness of mathematics and its importance.

For this purpose, we must strengthen all professional organisations of mathematical sciences and in particular strengthen Vijanana Parishad of India to play its due role in the development of applications of mathematics in India.

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भारतीय विज्ञान परिषद के कार्यवाहक:

अध्यक्ष डॉ. एस. एल. सिंह का उद्‍बोधन

आदर्णीय मुख्य अतिथि महोदय, प्रशासनिक समिति के अध्यक्ष श्री राघव जी, संस्था के प्राचार्य प्रो. सिलाकारी जी, रीवा वि. वि. के कुलपति प्रो. एम. के. जैन, संयोजक सचिव डॉ. आर. डी. अग्रवाल, परिषद सचिव डॉ. आर सी. सिंह चंदेल, परिषद के अन्य सम्मानीय सदस्यगण देश के विभिन्न भागों से पधारे सभी प्रतिनिधिगण, विदिशा के प्रबुद्ध नागरिकों, देवियों एवं सज्जनों :

आप सभी को विदित ही है कि किन्हीं अपरिहार्य कारणों वश परिषद के अध्यक्ष प्रो. जे. एन. कपूर यहां उपस्थित नहीं हो पाये हैं एवं उनकी अनुपस्थित हमें निरंतर खल रही है । यद्यपि उनके द्वारा प्रेषित वक्तव्य का वाचन डॉ. सक्सेना कर ही चुके हैं, फिर भी उनकी व्यक्तिगत अनुपस्थिति को किसी भी दृष्टिकोण से मैं पूरा नहीं कर सकता। मैं परिषद की ओर से पूरी ऊष्णता के साथ अपने प्रतिनिधियों का स्वागत करता हूं एवं मंत्री महोदय व सांसद महोदय का अभिनन्दन करता हूं कि उन्होंने अपने बहुमूल्य समय में से कुछ समय हमारे लिये निकाला है, साथ-ही-साथ उन्हें यह विश्वास दिलाता हूं कि वे निश्चित रूप से हमारी परिषद के बारे में अच्छी धारणा लेकर जायेंगे ।

प्राचार्य प्रो सिलाकारी जी ने अपने वक्तव्य में भारतीय विज्ञान और भारतीय मनीषियों की चर्चा की है वास्तव में हमारी भारतीय अस्मिता जिन मान्यताओं से जुड़ी हुई है, आधुनिकता की चकाचौंध में हम उनसे दूर होते जा रहे हैं । लेकिन आवश्यकतायें हमें उनकी याद दिला रही है ।

जैसा कि सभी जानते हैं कि मध्य प्रदेश की भूमि ने सदैव ही विद्वानों का आदर किया है । विद्वानों ने यहां पर अपना श्रेष्ठतम सृजन किया है। लब्ध प्रतिष्ठित वैज्ञानिक आर्यभट्ट, वराहमिहिर एवं भास्कराचार्य आदि की कर्मभूमि मध्य प्रदेश भी रही है । अतएव यह भूमि बंदनीय है और में उन पूर्व मनीषियों को प्रणाम के साथ ही अपनी बात दो शब्दों में कहना चाहूंगा ।

सर्वप्रथम भारतीय विज्ञान परिषद के उद्देश्यों पर प्रकाश डालना चाहूंगा । संस्थापकों ने परिषद के जो उद्देश्य रखे हैं, उनमें हमारी भावनाओं की झलक भी

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मिलती है। ये उद्देश्य अन्य गणितीय समितियों से भिन्न भी हैं। परिषद का मुख्य उद्देश्य भारत की ज्वलंत समस्याओं जैसे सामाजिक ग्रामीण, शिक्षा, एवं ऊर्जा सम्बन्धी समस्याओं से भारतीय विज्ञान परिषद को जोड़ना है। कल की कार्यकारी परिषद की बैठक में इन्हीं मुद्दों को ध्यान में रखते हुये एक प्रस्ताव भी पारित किया गया है, जिसके अनुसार इन उद्देश्यों की पूर्ति के लिये विज्ञान एवं प्रौद्योगिकी विभाग एवं अन्य बित्तीय संस्थाओं द्वारा संपर्क किया जाना है, ताकि परिषद अपने लक्ष्य प्राप्ति में सफल हो सके। यहां आकर मात्र यह कह देना कि हम राष्ट्रीय समस्याओं के प्रति जागरूक हैं, अपर्याप्त होगा। हमारे सामने बहुत पहिले से एक समस्या सामने खड़ी है, विज्ञान की शिक्षा के माध्यम की। भारतीय उच्च शिक्षा, विशेषरूप से विज्ञान व तकनीकी शिक्षा का माध्यम हमारी मातृभाषा नहीं है। कुछ समय पूर्व इन बातों को ध्यान में रखते हुये भारत शासन ने एक "तकनीकी शब्दावली मंडल" बनाया और उन्होंने एक "तकनीकी शब्दावली" की रचना भी की और यह आशा की गई कि विज्ञान की उच्चस्तर की पुस्तकें भी लिखी जायेंगी, तत्पश्चात मातृभाषा में विज्ञान पढ़ाने की सुविधा दी जायेगी। यहां एक व्यवहारिक कठिनाई आई कि जब तक हिन्दी में पठन-पाठन प्रारम्भ नहीं हो जाता, तब तक पुस्तकों का हिन्दी में प्रकाशन भी संभव नहीं, अलबत्ता कुछ अनुवाद अवश्य हो गये। इस दिशा में भारतीय विज्ञान परिषद कोई दावा तो नहीं करती किन्तु फिर भी इस बात पर विश्वास दिलाती है कि भविष्य में यदि हमारे प्रशासकों-राजनेताओं के सामने इस नीति के क्रियान्वयन में कोई परेशानी आती हैं। तो भा. वि. प. अपना सहयोग प्रदान कर महत्वपूर्ण भूमिका निभा सकती है।

आपको यह जानकर प्रसन्नता होगी कि इस सम्मेलन में भी दो-तीन शोध पत्रों के मुख्यअंश हिन्दी में प्राप्त हुये हैं और शोधार्थियों ने ये इच्छा व्यक्त की है कि उन्हें अपने शोध-पत्र हिन्दी में पढ़ने की अनुमति दी जाये। हमने इसे स्वीकार किया है।

हमारे देश के वैज्ञानिक-गणितज्ञ विदेशों की अंधी नकल पर इस बात को भूल जाते हैं कि जिन गरीब नागरिकों के कर से उन्हें जो तमाम सुविधायें मिलती हैं, उनके प्रति क्या उनकी कोई नैतिक जवाबदारी बनती है, उनके रहन सहन एवं पठन पाठन की सुविधाओं के प्रति हम कितने चिन्तित हैं, इन सब परिस्थितियों के बावजूद बड़े अफसोस के साथ कहना पड़ रहा है कि किसी भी सम्मेलन में हमारे गणितज्ञों-वैज्ञानिकों की नजर प्रथम श्रेणी के यात्रा भत्ता और पाँच सितारा सुविधाओं की ओर ही रहती है। वे इस बात को भूल जाते हैं कि हम एक गरीब देश के नागरिक हैं और न्यूनतम आवश्यक उपलब्ध सुविधाओं में हमें अपना कार्य करना है।

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आज आवश्यकता इस बात की है कि हमारे वैज्ञानिकों-गणितज्ञों का दृष्टिकोण मानवीय दर्दों की ओर आकर्षित किया जाये। उनको शोध-पत्र और प्रोजेक्ट्स के दायरे से बाहर आकर राष्ट्रीय समस्याओं (सामाजिक, शैक्षणिक, आर्थिक) के समाधान में कार्य करना है। भारतीय विज्ञान परिषद हर दृष्टिकोण से राष्ट्रीय समस्याओं के प्रति जागरूक है एवं हमारे प्रशासक राजनेता हमसे जिस प्रकार का भी सहयोग चाहते हैं, हम सदैव तत्पर हैं।

भारतीय विज्ञान परिषद भी इस क्षेत्र में अपनी ओर से कुछ पहल करेगी, आप सभी लोगों की भी इसमें महत्वपूर्ण भागीदारी होगी। इस भागीदारी को महत्वपूर्ण बनाने के लिये मेरा आपसे निवेदन है कि परिषद की सदस्यता को बढ़ायें एवं इसके उद्देश्यों पर प्रकाश डालकर लोगों को सदस्य बनने हेतु प्रेरित करें। इन शब्दों के साथ एक बार पुनः अतिथियों का हार्दिक अभिनंदन करता हूं कि उनकी उपस्थिति से हमारा मनोबल बढ़ा है।

धन्यवाद। जय हिन्द।

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मुख्य अतिथि माननीय श्री महेश प्रसाद जी शुक्ला जनशक्ति नियोजन मंत्री, मध्य प्रदेश शासन का उद्बोधन

आदरणीय अध्यक्ष महोदय, महाविद्यालय के प्राचार्य महोदय, विभिन्न स्थानों से आये आदरणीय वैज्ञानिक बन्धुओ इस नगरी के विद्वज्जन एवं बहिनों ।

मुझे यह जानकर बड़ी प्रसन्नता हो रही है कि "भारतीय विज्ञान परिषद" गणित एवं विज्ञान की प्रगति जैसे महान कार्य में लगी हुई है । जिस महान कार्य की दिशा में "भारतीय विज्ञान परिषद" का यह अधिवेशन हो रहा है, उस कार्य की दृष्टि से मैं अल्पज्ञ हूं । अतएव आपको उद्बोधन करूं अथवा मार्गदर्शन करूं, ऐसी सामर्थ्य का मैं अपने में नितान्त अभाव अनुभव कर रहा हूं ।

मेरी अपेक्षा यह है कि, थोड़ा-सा ही सही, आपसे यदि कुछ पा सका तो मेरा आज यहां आना सार्थक होगा । अपने पूर्व वक्ताओं से मुझे यहां बहुत सी बातें सुनने को मिली हैं । इन्हीं चर्चाओं से मार्गदर्शन प्राप्त करते हुए, मैं संक्षिप्त में अपनी बात आपके सामने रखूंगा ।

यह बिल्कुल सनातन-सत्य है कि चाहे विद्या का क्षेत्र हो, साहित्य का क्षेत्र हो अथवा विज्ञान का क्षेत्र हो, इस देश ने प्रगति की पराकाष्ठा प्राप्त की थी । समूचे विश्व में "ज्ञान गंगा" के प्रवाह का उद्गम हमारी मातृभूमि में था । समय के अनुरूप ही समाज और राष्ट्र के जीवन में बदलाव एवं उतार-चढ़ाव आते हैं । ऐसी ही एक परिस्थिति, विगत कुछ बर्षों में हमारे देश में निर्मित हुई । जबकि हमारे स्वदेशी बन्धुओं में इस भावना ने जन्म लिया कि "ज्ञान प्रगति अथवा शोध" सिर्फ पश्चिमी राष्ट्रों में ही है और जो कुछ हमारा है उसमें कोई तत्व ही नहीं है "वह हेय है । अतएव वह त्याज्य है और जो कुछ पश्चिम में हो रहा है वह अनुकरणीय है । इस अंधानुकरण का प्रवाह इतना तेज था कि उसमें हमारा सब कुछ बह गया ।

यह सर्वविदित है कि भारत वर्ष में प्रतिभाओं का कभी भी अभाव नहीं रहा है । लेकिन समय की आंधी में हमारी प्रतिभायें भी विचलित हुईं और उसके दुष्परिणाम सदियों तक हमारे इस महान् देश को भोगना पड़े ।

आज यहाँ हम सभी "भारतीय विज्ञान परिषद" के द्वितीय वार्षिक अधिवेशन में सम्मिलित हुए हैं । मुझे बड़ी प्रसन्नता है कि यह परिषद देशी प्रतिभाओं के विकास की दिशा में भी प्रयासरत है । मेरे पूर्व वक्ताओं ने साधनों, वित्तीय साधनों एवं अवसरों के सबंध में बहुत सी बातें सामने रखी हैं । मैं म०प्र०शासन की ओर से केवल ये निवेदन करना चाहता हूं कि वह आपके विचारों से पूर्णतः सहमत है । हमारी यह मान्यता है कि

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गणित ही "मूलभूत विज्ञान" है । अतः ज्ञान अथवा शिक्षा के किसी भी क्षेत्र में विकास हेतु गणित के क्षेत्र में विकास आवश्यक है, आज देश के बहुमुखी विकास के लिये जो प्राथमिक आवश्यकता है, वह है गणित के अध्ययन की ओर गणित-विज्ञान में दक्षता की । ज्ञान का ऐसा कोई भी क्षेत्र नहीं है जो गणित शास्त्र अथवा गणित विज्ञान के बिना चल सकता हो । अतः विज्ञान परिषद प्रथमतः जो गणित के विकास हेतु प्रयासरत है, उसके लिये मैं उनका हार्दिक स्वागत करता हूं, साधुवाद देता हूं । और साथ ही साथ परमपिता परमेश्वर से प्रार्थना करता हूं कि इन साधकों को यश-साधन-सामर्थ्य दें, ताकि वे अपने इस महान कार्य में सफल हों ।

मैं म० प्र० शासन की ओर से आपसे निवेदन करना चाहता हूं कि आप हमसे जिस सहयोग की अपेक्षा करते हैं, अपने सीमित साधना के अनुरूप शासन अपने पूरे सामर्थ्य के साथ आपको सहयोग करेगा । किंतु आपसे अनुरोध यह है कि जो थोड़ा बहुत भी सहयोग शासन से प्राप्त हो उसे "सुदामा के तंदुल" एवं "शबरी के बेर" मानकर सहर्ष स्वीकार करें और आपके विकास की जो "पावन यात्रा" है, उसमें म० प्र० शासन भी सहभागी बने ।

मैं एक बात पर आप सभी का ध्यान पुनः आकर्षित करना चाहूंगा कि आज भी देश में ऐसे नागरिक मौजूद हैं, जिनको दो वक्त का भोजन भी उपलब्ध नहीं हो पाता है । अतः देश का समग्र विकास गरीबी की रेखा के सबसे अन्त में खड़े उस नागरिक से आंका जाना चाहिये, जिसे दो वक्त की रोटी भी मुहैया नहीं हो पाती । इसी गरीब नागरिक के जीवन में हम कितना सुख अथवा कितनी समृद्धि ला सकते हैं, यही हमारी प्रगति का माप दण्ड होना चाहिये । मेरा आपसे निवेदन है कि आपके मार्गदर्शन में प्रदेश में अथवा देश में जो भी योजनायें चल रही हों उनका लक्ष्य 'अन्त्योदय' होना चाहिये ।

मेरी और म० प्र० शासन की ये आंकाक्षा है कि इस 'यज्ञकार्य, में लगे हमारे इन साधकों को इस देश की प्राचीनतम परम्पराओं के अनुसार वह गौरव व मान सम्मान मिले जो हमारे ऋषियों और मनीषियों को प्राप्त था । अतीत में हमारे देश की प्रगति का कारण उस समय की परम्परायें मानदण्ड व मूल्य थे । आज पुनः इस देश की प्रगति के लिये आवश्यक हो गया है कि जीवन के मानदण्डों, जीवन के मूल्यों को पुर्नस्थापित किया जाये । हमारी परम्परा रही है कि जिसने जितना अधिक छोड़ा है, वह उतना ही महान् बना है और जिसने जितना अधिक जोड़ा है वह उतना ही अधिक तुच्छ माना गया है । अब समय आ गया है जबकि हम सभीको अपने प्राचीन मापदण्डों एवं मूल्यों को पुर्नस्थापित करने के लिये उतनी ही सक्षमता एवं ईमानदारी से प्रयास करना है जितना कि हम वैज्ञानिक प्रगति के लिये प्रयासरत हैं । इन थोड़े से शब्दों के साथ आपका पुनः आभार मानते हुए, अपनी असमर्थता को अनुभव करते हुए "मैं आज भारतीय विज्ञान परिषद के इस द्वितीय वार्षिक अधिवेशन को उद्घाटित घोषित करता हूं" । जय भारत ।

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समारोह के अध्यक्ष माननीय श्री राघव जी भाई, सांसद एवं सचिव महाराजा जीवाजीराव एजुकेशन सोसायटी, विदिशा का उद्बोधन

आज के समारोह के मुख्य अतिथि आदरणीय श्री शुक्ला जी, इस संस्था के प्राचार्य श्री सिलाकारी जी, रीवा वि० वि० के उपकुलपति श्री जैन साहब, भारतीय विज्ञान परिषद के कार्यवाहक अध्यक्ष एव अन्य पदाधिकारीगण, मुरैना के सांसद श्री छविराम जी अर्गल, सोसायटी के उपाध्यक्ष श्री जालौरी जी, श्री राजेन्द्र जी शर्मा, श्री पवार साहब, उपस्थित संभ्रान्त नागरिक बन्धुओं दूर-दूर से पधारे वैज्ञानिक वंधुओं और बहिनों:

आज हमारी संस्था गर्व गौरव अनुभव कर रही है कि विज्ञान परिषद के वार्षिक अधिवेशन के अवसर पर देश के कोने-कोने से वैज्ञानिक एवं गणितज्ञ इस प्रांगण में पधारे हैं। साधनों की कमी होने के कारण संभव है कि उनके स्वागत सत्कार की अच्छी व्यवस्था न हो पायी हो अतएव मैं क्षमाप्रार्थी हूं। चूंकि आप एक महान कार्य के लिये यहां पर एकत्रित हुये हैं, अतः मैं आपसे अपेक्षा करता हूं कि आप इन किंचित असुविधाओं की ओर ध्यान नहीं देंगे।

भारत का अतीत बड़ा गौरवमय रहा है एवं विज्ञान के क्षेत्र में भारतवर्ष सदैव अग्रणी रहा है। अतीत में जिस समय हम विज्ञान के क्षितिज पर थे, अन्य देश विज्ञान के धरातल पर भी नहीं थे। किन्तु आज परिस्थितियां एक दम विपरीत हैं आज हम विज्ञान के क्षेत्र में अन्य देशों की तुलना में बहुत पिछड़े हुए हैं। इस पिछड़ेपन के दर्जनों कारण हो सकते हैं, किन्तु ये कारण कदापि सभव नहीं है कि हमारे देश में प्रतिभाओं का अभाव है। इस देश में प्रतिभाओं का कभी भी अभाव नहीं रहाहै। वर्तमान में भी हमारे देश के वैज्ञानिक व चिकित्सक विदेशों में जाकर प्रसिद्धि पा रहे हैं। अतः यहाँ पर प्रश्न प्रतिभाओं के अभाव का न होकर इस बात का है कि हमारे वैज्ञानिकों -चिकित्सकों को कितने अवसर व कितनी सुविधायें मिल रहीं हैं, अपनी प्रतिभा निखारने के लिये?

वर्तमान में हम विज्ञान के क्षेत्र में तो बहुत पिछड़े हैं ही साथ ही साथ हमारी अर्थ व्यवस्था भी चरमरा रही है। हम विदेशी कर्ज के बोझ से दबे जा रहे हैं। आज हमारे देश के ऊपर लगभग दो लाख अस्सी हजार करोड़ रु० का कर्ज है। इस कर्ज के ब्याज के रूप में हमको लगभग सत्रह करोड़ रु० चुकाना पड़ते हैं, जो कि

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हमारे सुरक्षा बजट से कहीं अधिक है । आज हम आयात-निर्यात के व्यापार में लगभग सात हजार करोड़ रु० के घाटे में चल रहे हैं । सिर्फ पेट्रोलियम पदार्थों का आयात ही लगभग पांच हजार करोड़ रु० का होता है । अतः आज इस बात की परम आवश्यकता है कि आयात-निर्यात के बीच संतुलन बनाया जाये । जब तक हम ये संतुलन नहीं बनायेंगे तब तक हमारा देश उबर नहीं सकता है । ये चुनौतियां हैं, हम सब के सामने, हमारे वैज्ञानिकों गणितज्ञों के सामने, हमारी प्रतिभाओं के सामने ।

आज हमारे देश में प्रतिभायें मौजूद हैं, लेकिन फिर भी हम पेट्रोलियम पदार्थों व अन्य खनिजों का उत्खनन सही मात्रा व गुणवत्ता में क्यों नहीं कर पा रहे हैं ? अच्छी से अच्छी धातुयें व लोहे का निर्माण हमारे देश में क्यों नहीं हो पा रहा है ? ये ज्वलंत प्रश्न हैं, आपके सामने । इन समस्याओं का हल आपको निकालना है । क्योंकि इन समस्याओं को हल किये बगैर अतीत के वैभव को प्राप्त नहीं किया जा सकता है । आज भाषणबाजी की आवश्यकता नहीं है, वरन् समय की मांग है कि "काम करो"।

अतीत में हमारे वैभव का कारण हमारे ऋषियों-मुनियों की सतत-साधना थी । उनकी पीढ़ी दर पीढ़ी इस साधना में समाप्त हो गई, किन्तु फिर भी उन्होंने अपनी या अपने परिवार की सुख सुविधाओं की चिन्ता नहीं की और अपने सोचे गये लक्ष्य के प्रति समर्पित रहे । आज वही समर्पण का भाव जाग्रत करने की आवश्यकता है ।

आज किसी भी मिशन में, किसी भी लक्ष्य प्राप्ति हेतु वित्तीय ससाधनों की आवश्यकता होती है, इस बात में कोई सदेह नहीं है । अतः जो कुछ भी संभव हो सरकार जुटायें । आर्थिक संसाधन उपलब्ध कराना उनका परम कर्त्तव्य भी होना चाहिये, क्योंकि इसका कोई विकल्प नहीं है, कोई अन्य रास्ता नहीं है । सभव है कि शासन द्वारा उपलब्ध कराये गये आर्थिक संसाधन पर्याप्त न हों, किंतु फिर भी हमारे वैज्ञानिकों को समर्पित भाव से जूझना पड़ेगा । ये भारत माता की पुकार है और मुझे पूर्ण विश्वास है कि भारत माता के ये सपूत इस आव्हान को व्यर्थ नहीं जाने देंगे । इसके लिये हमें जो भी कष्ट सहना पड़ेगे हम सहेंगे । जो भी त्याग करना पड़ेगा हम करेंगे इसके सुखद परिणाम कुछ ही वर्षों में दिखेंगे । हमें भारतवर्ष प्रगति पथ पर अग्रसर दिखेगा ।

हमारे देश की धरती धन-धान्य, खनिज संपदा और विभिन्न प्रतिभाओं से परिपूर्ण है । इतना सब कुछ होते हुये भी हमें भीख का कटोरा लेकर दूसरे देशों के पीछे घूमना पड़ता है । हमें छोटी-छोटी सी चीजें आयात करने के लिये विदेशों का मुंह देखना पड़ता है । ये जो कुछ भी चुनौतियां आपके सामने हैं, उसमें आपकी क्या भूमिका हो सकती है, आप स्वयं निर्धारित करें ।

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परिषद के कार्यवाहक अध्यक्ष जी ने अपने संबोधन में कहा था कि "हमारी विज्ञान परिषद भारतीय भाषाओं को शिक्षा का माध्यम बनाने में सहायता कर सकती है"। मुझे बहुत प्रसन्नता हुई। भारतीय भाषाओं को विज्ञान क्षेत्र में माध्यम बनाने में बहुत सी कठिनाईयां हैं। पुस्तकें पहिले लिखी जायें कि पठन-पाठन पहिले प्रारंभ किया जाये ? ये ही गुत्थी नहीं सुलझ पायी है। अतः हम उस दिशा में आगे नहीं बढ़ पा रहे हैं। आप लोग इसका रास्ता खोजिये, आपने इसका वायदा भी किया है। इसके लिये राज्य सरकारों व केन्द्र सरकार की सहायता भी ली जा सकती है। ये तो एक छोटा सा संकल्प आपने यहां पर व्यक्त किया है, ऐसे कई और संकल्प लिये जा सकते हैं और संकल्प व्यक्त न करते हुये भी कई महत्वपूर्ण उपयोगी कार्य किये जा सकते हैं। मुझे विश्वास है कि आने वाले दो दिनों में यहाँ पर जो चिंतन होगा, जो चर्चायें होंगी और जो विषद्व्याख्यायें होंगी उनमें इस पृष्ठभूमि को भी आप ध्यान में रखेंगे।

अन्त में एक बार पुनः इस ऐतिहासिक नगरी विदिशा मे आप सभी का हार्दिक स्वागत करता हूं।

धन्यवाद !

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UNDERLYING MATHEMATICAL ELEMENTS OF SOME AI APPLICATIONS

By

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A general definition of AI, to start with, can be given as:

"AI is the computer based solution of complex problems through the application of processes that are analogous to the human reasoning process."

Since the problems in AI domain call for solutions that involve 'reasoning type of activity' the solution development process i. e. the methodology turns out to be different as compared to the conventional programming problems. The components of AI-problem-solution may be described in following steps:

(i) Formal representation of problem e.g. state space representation.

(ii) Selections of search method e.g. breadth first search, depth-first search, best first and blind search.

(iii) Controlling the direction of search e.g. forward or data driven reasoning, backward or goal directed reasoning.

(iv) Applying knowledge to guide the search i.e. use of heuristic knowledge to increase the efficiency of a reasoning process by providing direction and thereby reduce the state space that must actually be searched before a solution isfound,

A simple AI problem can now be looked into to appreciate some of the points mentioned above.

The two-pail problem:

Two empty water pails are given, one with a capacity of 6 gal and the other with a capacity of 8 gal. Given that either pail can be

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filled in at will, how to get the 8-gal pail exactly half full ?

A general approach to solve above problem is to first identify possible sequences of actions and conditions wnen a particular action could be initiated. Any action would alter state of the two pails. The problem thus can be viewed as the one in which a sequence of states muss be found that leads to particular state of two pails, in this case a state in which 8 gal pail is half full and the 6 gal pail may or may not contain water. The collection of all discrete states is called state space. The solution is thus obtained by searching of the state space to find a valid sequence of states leading to a desired state. The operations and a reduced state space is given below.

Operations:

Action Number	Action	Condition for application
1	Fill the 8-gal pail.	8-gal pail is not full.
2	Fill the 6-gal pail.	6-gal pail is not full.
3	Dump the 8-gal pail.	8-gal pail is not empty.
4	Dump the 6-gal pail.	6-gal pail is not empty.
5	Empty the 8-gal pail into the 6-gal pail.	6-gal pail is not full, and the 8-gal pail not empty. The combined content is <=6 gal.
6	Empty the 6-gal pail into the 6-gal pail.	8-gal pail is not full, and the 6 gal pail is not empty. The combined content is <=8gal.
7	Fill the 8-gal pail from the 6-gal pail.	8-gal pail is not full, and the 6-gal pail is not empty. The combined content is=>8gal.
8	Fill the 6-gal pail from the 8-gal pail.	6-gal pail is not full, and the 8-gal pail is not empty. The combined content is=>6 gal.

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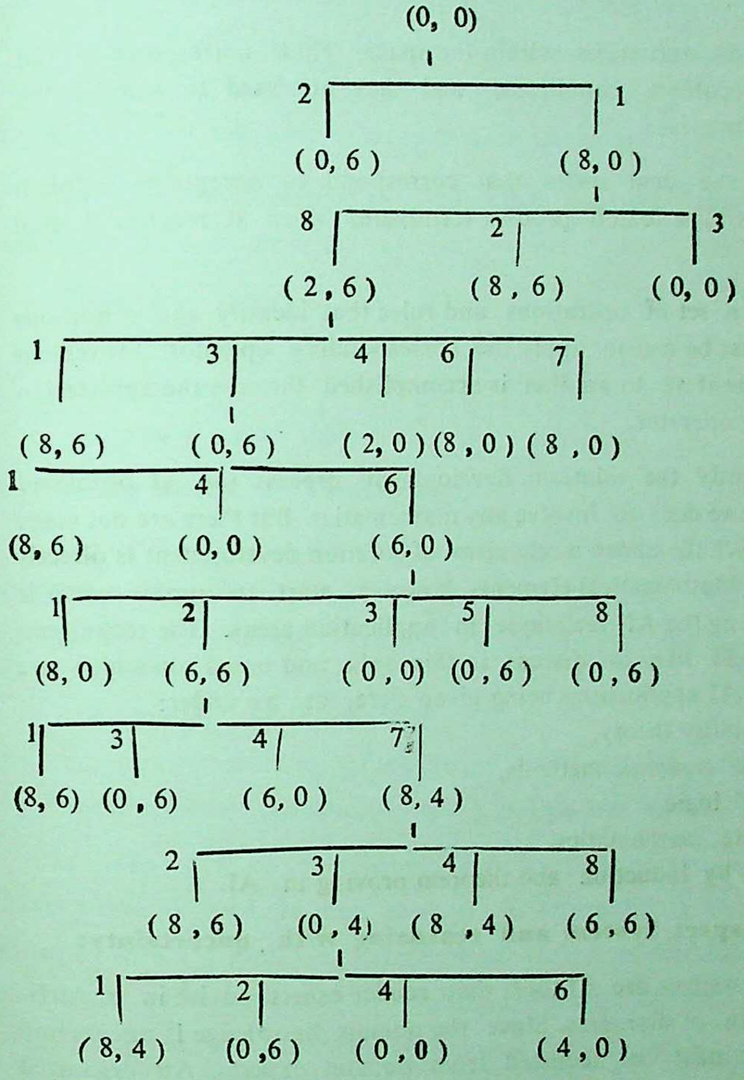
The reduced state space:



The numbers along vertical lines represent action number, and numbers within brackets represent states of the two pails.

A more formal description of the process for devaloping the state –space representation, thus, could be:

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1. Define the problem environment as a collection of states in which each state corresponds to a unique configuration of the domain elements.

2. Define the start states within the space. These correspond to the initial problem conditions, and they are used to initiate the search process.

3. Define the goal states that correspond to acceptable problem solution. The search process terminates when it reaches a goal state.

4. Define a set of operations and rules that identify the conditions that must be met to apply the corresponding operator. Movement from one state to another is accomplished through the application of one operator.

Apparently the solution development process for AI problems as stated above does not involve any mathematics. But there are not many areas in which the above mechanism of solution development is directly applicable. Mathematical elements, however, start to appear when it comes to using the AI technique in application areas. The techniques that we would like to discuss in this talk and based on which are some of the AI applications being given here, are as under:

- probability theory,
- simple algebraic methods,
- formal logic,
- discrete mathematics,
- proof by induction and theorem proving in AI.

The Expert System and reasoning with uncertainty:

Expert systems are AI tools that render expert advise in a particular domain of discourse. Since the domain knowledge is uncommon in nature it must be acquired from domain experts. An organized form of this knowledge is made available for being accessed for drawing of inferences. In several applications it is seldom or rare that an expert can be one hundred percent confident of validity of knowledge he posses. Similarly, the person asking for advise may not be 100 percent sure about all of the inputs he provides to the expert

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during consultation. All of these uncertainties are factored into the human reasoning process. As a aesult, the advise that the expert gives may be qualified by some degree of certainty. AI techniques used for factoring varying levels of certainty into the reasoning performed by an expert system use various mathematical techniques.

There are several method to handle the uncertainty in knowledge and data. Some of these are.:

- probability theory
- certainty factor method
- Fuzzy logic
- Dempster-Schafer theory

Reasoning Based on Probability:

The actions of an expert system are typically the result of piecing together many different fragments of knowledge, each with different probability characteristics. We must be able to combine probabilities. For example:

$$P(E_1 \text{ and } E_2) = P(E_1) * P(E_2)$$

gives combined probability of occurring two separate events E_1 and E_2 if the individual probabilities of E_1 and E_2 are known.

In case of more complicated systems like diagnostic domains Bayes's rule is used. In such cases if B_i represents a cause of the effect E, then $P(B_i/E)$ is inferred belief that B_i is the cause of effect E. We can use this relation to quantify the extent to which we would believe B_i given the available evidence E.

The Bays's rule states:

$$P(B_i / E) = \frac{P(E / B_i) * P(B_i)}{\sum_{j=1}^{k} P(E / B_i) * P(B_i)}$$

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Certainty Factors (CF) :

The concept of CF was developed for Mycin. A certainty factor (CF) is relatively informal mechanism for quantifying the degree which, based on the presence of a given set of evidence, we believe (or disbelieve) a given conclusion.

Calculation of Certainty factors :

The cumulative certainty factor is formed by combining global degrees of belief and disbelief represented by the 'cumulative measure of belief' and the 'cumulative measure of disbelief' for the conclusion.

Specifically, a cumulative certainty factor is defined, for a specified point during system execution, as follow:

$$CF[c, e_c] = MB[c, e_f) - MD[c, e_a]$$

where

c : := the conclusion under consideration

e_c : := all of the evidence that relates to c that has been considered upto the specific point of execution

$CF[c, e_c]$: := the cumulative certainty factor for c given e_c i.e. the net belief in the conclusion, given the current evidence

e_f : := all of the evidence 'for c' that has been considered

e_a : := all of the evidence 'against c' that has been considered

$MB[c, e_f]$: := cumulative measure of belief for c given e_f

$MD[e, e_a]$: := cumulative measure of disbelief for C given e_a

The cumulative measures of belief or disbelief involve expressions. The methods of computation of joint certainty factor for such an expression are founded in Algebra. Some such methods are minimum method, product method, average method, Bonczek-Eagin method etc . . These methods are applicable in cases where certainty factors are connected using 'AND' conjunction. In cases where 'OR' conjunction is used to combine CFs in an expresion methods, which are applicable, are maximum method, probability sum method,average method etc .. The method stated above have been implemented in the Guru expert system.

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Formal log'c:

Formal Logic is one of the earliest forms of knowledge representation. In this prepositional and predicate forms are two widely used methods.

Consider following statement:

"If the parliament refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the president of the firm resigns."

If we represent the facts stated above as follows:

the parliament refuses to act by r,
the strike is over by O,
the president resigns by P, and
the strike lasts more a year by Y,

then the complete statement can be written as

$$r \Rightarrow (\neg O \vee (Y \wedge P)),$$

which reads: r implies (not O or (Y and P)).

The entity stated above is called 'formula' in prepostional logic. In logic, using well founded theorems and relations, such formulae are solved by subjecting them to various transformations, to deduce that they a e logical consequence of a set of given formulae and are either valid or inconisistent.

In predicate logic, the First order logic in particular, in this case we use a different logic formalism to represent statements for performing inferences on them. Consider the statement:

DSK is a man and all men are mortal.

From this statement we want to infer that 'DSK' is mortal'. This in FOL, is represented as

$$\forall X (\text{human} (X) \Rightarrow \text{mortal} (X)$$

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to say that for all X if X is a humen then X is mortal. Here 'human' and 'mortal' are predicate symbols of arity 1 and X is a variable which is universally quantified.

In FOL a formula is true if it evaluates to 'True' under all interpretations over all domains. If a formula evaluates to False under all interpretations then the formula is unsatisfiable or inconsistent.

Unlike propositional logic, verfication of the validity of a formula in FOL by enumerating all possible interpretations and examining the truth value of the formula in each of the interpretation is impossible. This necessitates transformation of FOL formula in various forms. This sound mathematical foundation of predicate logic in general and FOL in particular has made it befitting tool which has led to the development of several logic programming systems e.g. PROLOG for implementation of AI problems.

Discrete mathematical elemetns:

In areas like computational linguistics or musicology the need to describe inherent grammar and on its basis the language is obvious.

In this section we present the mathematical symbolism to represent production rules, sentential forms and language definition for a special subclass of gramma s called replacement sysstems.

A production rule has the general form $X \rightarrow Y$, where X and Y are strings in terminal and nonterminal sets of a given grammar.

In such a system, we can generate other strings in the language by starting with a string of the form *wxz* then applying a production rule of the form $x \rightarrow y$ to yield a new string *wxz*. Ttus wxz= wyx called a deviation step is represented using symbol $\Rightarrow$.

Each of the strings wxz and wyz are called 'sententi al forms' provided that we started with the start symbol S of the grammar and obtained wxx through a sequence of derivation steps.

A sequence of one or more derivation steps is indicated by $\overset{+}{\Rightarrow}$.

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For example, in grammar G_0, we have:

$E \Rightarrow E+T \Rightarrow E+T+T \Rightarrow E+T+T+T$; then we may write $E \Rightarrow^{+} T+T+T$

Here since E is the start symbol, the string $T+T+T$ is a sentential form in G_0.

A sentential form consisting of only terminals is called a sentence. If we have $S \Rightarrow^{+}$ { some sentence } then we have derived a member of the language defined by the grammar. We may put this in set notation as follows:

Given a grammar $G = (N, \Sigma, P, S)$, then
$L (G)$, the language defined by G is :

$L (G) = \{ x | \ x \in \Sigma^*, \text{ and } S \Rightarrow^{+} x \}$

The form $\{ x/c \}$ is read "the set of all x's such that condition c holds". $L(G)$ is then the set of all strings x such that x is terminal and x can be derived from the start symbol S.

Mathematical theorem proving is yet another area in which AI joins hands with mathemetical principle of induction. The list of such areas is large and uses of mathematics in AI application are enormous, and therefore we have only touched upon, those which are in wide use. It is thus seen that though mathematical elements have a limited direct role in AI technique, as such, it is, however deep rooted within the AI applications to a very large extent so much so that without this thinking of any sophisticated AI technique worth being called 'intelligent' is difficult, if not impossible.

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SPRINGS, SAVIOUR, AND PHILOSOPHY OF THE HERMITE POLYNOMIALS

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ABSTRACT

We present various starting points for developing the properties of the Hermite polynomials. We show that each and every Hermite polynomial will be a simple polynomial if the constant term of a single Hermite polynomial of even degree is non-zero. Using the Wronskian, we present new proofs of the definite parity of the Hermite polynomials and the recurrence relations satisfied by them. We point out that intereseing lessons can be learnt from the properties of the Hermite polynomials.

1. Introduction

The purpose of this talk is to present the following relation

$$H_n(x) = g^{-1}\,[\,2x - D + g^{-1}\{Dg\}\,]^n\, g, \quad D \equiv d/dx, \quad d \leqslant 0.1$$

where $g(x)$ is *any* differentiable function not identically zero, as the starting point (spring) of the starting points from which one can develop the properties of the Hermite polynomials [1—12]. (We follow the convention that $R^0 = I$, the unit operator, for any operator R.) We next show that *all* the Hermite polynomials will be simple polynomials if the constant term of *any* Hermite polynomial of even degree is non-zero. Using the Wronskian, new proofs of the definite parity of the Hermite polynomials and the recurrence relations satisfied by them are given. We also present the philosophical side of the Hermite polynomials.

2. Springs of the Hermite Polynomials

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In Ref. [10] we have begun with a derivation of (1) and then shown that all the properties of the Hermite polynomials and many a beautiful relation follow from it. The interesting point about (1) is that one need not specify what $g(x)$ is. *Any* differentiable function not identically zero will snffice. Thus, for example, when $g = 1$, we obtain the beautiful relation given in Arfken [2, Prob. 13. 1. 5, P 718]:

$$H_n(x) = (2x - D)^n\, 1, \quad n \geqslant 0. \qquad (2)$$

When $g = \exp(-x^2/2)$, we derive the relation

$$H_n(x) = \exp(x^2/2)\,(X - D)^n \exp(-x^2/2), \; n \geqslant 0, \qquad (3)$$

a result which is very useful in the quantum mechanical treatment of a simple harmonic osillator (see, e. g., [2, 13]). When $g = exp(-x^2)$, we deduce from (1) the Rodrigues' representation

$$H_n(x) = (-1)^n \exp(x^2)\, D^n \{\exp(-x^2)\}, \; n \geqslant 0. \qquad (4)$$

It is now clear that the spring of springs (i. e. (1)), the Rodrigues' representation (i. e, (4)), Arfken's formula (i. e. (2)) and (3) are all equivalent.

Relation (1) has been obtained as a natural consequence of the standard properties of the Hermite polynomials [10]. We have shown that (1) is a spring for developing the properties of $H_n(x)$ [10]. The following relation

$$H_n(x) = (2x - D)\, H_{n-1}(x), \; n \geqslant 1, \qquad (5)$$

plays a crucial role in establishing the results that (1) and (2) are springs of the Hermite polynomials [9, 10].

We can prove that the following interesting relation from Bell [3, Theorem 5 . 3 *p.* 159]

$$H_n(x) = 2^n \{ \exp(-D^2/4)\, x^n, \quad n \geqslant 0, \qquad (6)$$

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and the power series expansion of the Hermite polynomials

$$H_n(x) = \sum_{s=0}^{[n/2]} \frac{(-1)^s \, n! \, (2x)^{n-2s}}{s! \; (n-2s)!}, \quad n \geqslant 0, \tag{7}$$

where $[r]$ is the greatest integer $\leqslant r$, are really equivalent.

The starting points can be clearly classified into two distinct groups: (*a*) full/complete springs and (*b*) associate (incomplete or partial) springs. To the first category belong the generating function

$$\exp(2tx - t^2) = \sum_{n=0}^{\infty} H_n(x) \, t^n / n! \, , \tag{8}$$

the Rodrigues' reprcsentation, the power series expansion, relations (1) - (3) and (6), and the orthonormal property

$$\int_{-\infty}^{\infty} \exp(-x^2) \, H_m(x) \, H_n(x) \, dx = 2^n \, n! \, \sqrt{\pi} \, \delta_{mn}. \tag{9}$$

These springs specify the Hermite polynomials completely. The differential equation *(DE)*

$$(D^2 - 2xD + 2p) \, H_p(x) = 0, \quad p = 0, 1, 2, \ldots, \tag{10}$$

the pure and diferential recurrence relations

$$H_{n+1}(x) = 2x \, H_n(x) - 2n \, H_{n-1}(x), \quad n \geqslant 1, \tag{11}$$

$$D \, H_p(x) = 2p \, H_{p-1}(x), \quad p \geqslant 1, \tag{12}$$

the orthogonal property and *(5)* belong to the second category as they require supplementary conditions to specify the Hermite polynomials fully. The constant term of any $H_n(x)$, $n \geqslant 1$, cannot be found from the differential recurrence relation as the hungry operator D simply swallows it. The normal choice for this term is given by

$$H_{2m+1}(0) = 0, \quad m \geqslant 0, \tag{13}$$

$$H_{2m}(0) = (-1)^m \, (2m)! \, / \, m!, \quad m \geqslant 0. \tag{14}$$

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In the case of the orthogonal property we require the value of the right hand side of (9) when $m = n$, for all $n \geqslant 0$ (the square root of the reciprocal of this quantity is the so-called normalization constant), and to make (5) a complete spring we require the result $H_0(x) = 1$.

An outline of the development of the various properties from the springs is shown schematically in Fig. 1. (Of course, not all the paths are shown.) Certain properties can be more easily obtained from a given spring, while it may be tedious to derive another property from the same spring. The method of induction is very useful in developing certain properties from a given starting point. Some properties can be independently obtained from a given spring without going either via the generating function or via the Rodrigues' representation.

Criticisms can be levelled against any spring. For Simmons [5, *p.* 189], the generating function method is totally unmotivated though it has the advantage of efficiency for deducing the properties of the Hermite polynomials. While he develops the properties from the differential equation, Andrews [1, *p.* vii] introduces the classical orthogonal polynomials by the generating function method and Rainville [4] revels in the generating function approach. Relation (2) is simple and handy, but may have the obvious weakness of being completely unmotivated.

3. Saviour of the Hermite Polynomials

Next we look at the significance of the constant term of *any* Hermite polynomial of even degree. If $H_{2n}(0)$ is non-zero for a *single* positive integer $n \geqslant 0$, the entire set of the Hermite polynomials $\{ H_m(x) \}$, will form a simple set of polynomials, each polynomial in the set having a non-zero leading term and hence a precise degree [4, 6].

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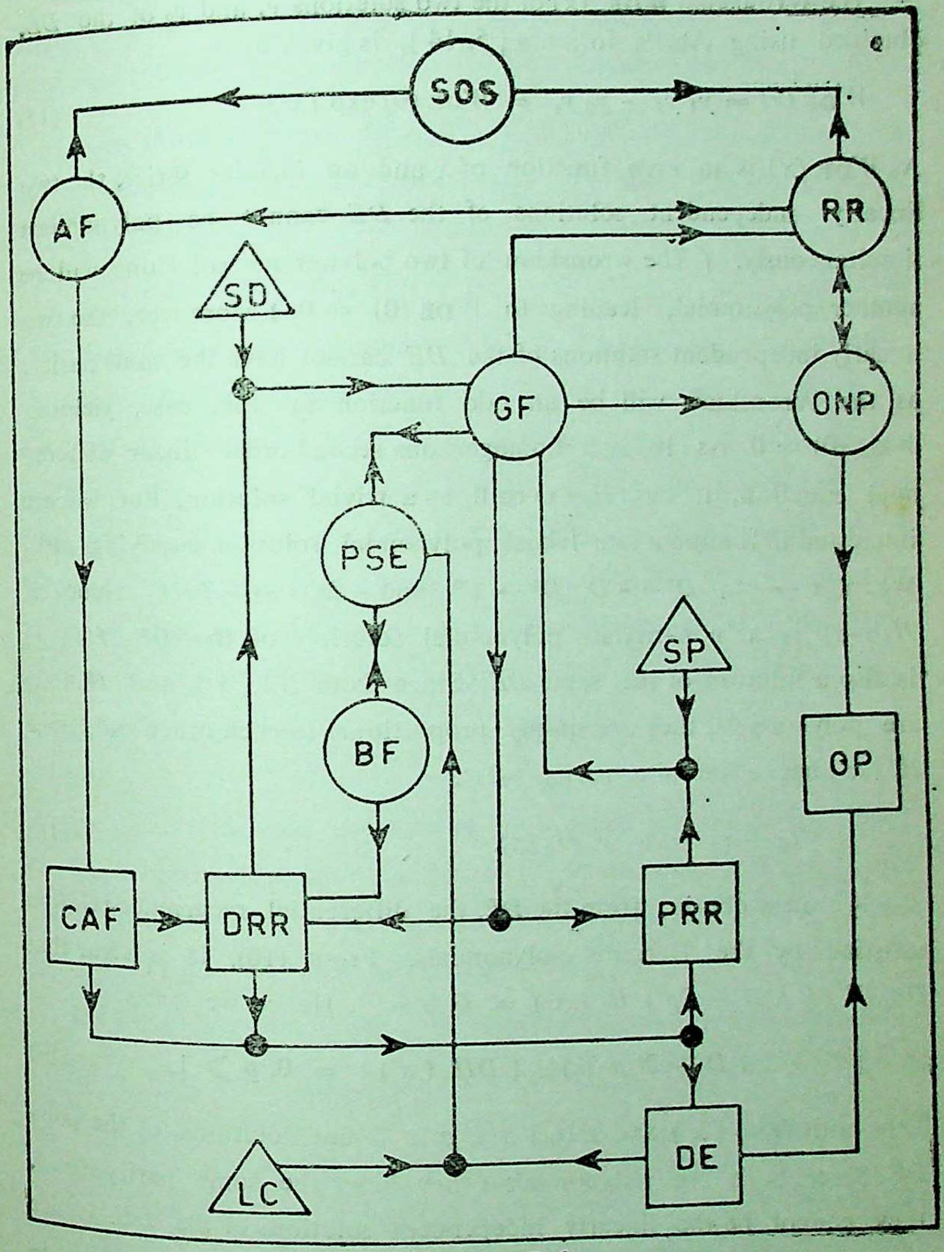

FIGURE 1.

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The Wronskian $W_{DE}(x)$ of the two solutions y_1 and y_2 of the DE, obtained using Abel's formula [5, 14], is given by

$$W_{DE}(x) = y_1 y_2' - y_2 y_1' = W_{DE}(0) \exp (x^2). \tag{15}$$

As $W_{DE}(x)$ is an *evcn* function of x and an *infinite* series, the two linearly independent solutions of the DE cannot be polynomials simultaneously. (The wronskian of two polynomial solutions will be another polynomial, leading to $W_{DE}(0) = 0$.) Moreover, the two linearly independent solutions of the DE cannot have the same parity, as the Wronskian will be an odd function in this case, yielding $W_{DE}(0) = 0$. As (10) is a homogeneous second order linear differential equation, it has $H_p(x) \equiv 0$ as a trivial solution. But we are interested in finding a non-trivial polynomial solution satisfying (10). When $x \to -x$, $D \to -D$, $D^2 \to D^2$, and $-2xD \to -2xD$. Hence if $H_p(-x)$ is a non-trivial polynomial solution of the DE, $H_p(-x)$ is also a solution of the same DE. Since both $H_p(x)$ and $H_p(-x)$ are polynomials, they are simply proportional to each other and hence $H_p(x)$ has a definite parity $(-1)^p$:

$$H_p(-x) = (-1)^p H_p(x). \tag{16}$$

We next obtain, from the DE, the differential recurrence relation satisfied by the Hermite polynomials. From (10), it is clear that $D(D^2 - 2xD + 2p) H_p(x) = D\,0 = 0$. Hence we have

$$\{ D^2 - 2xD + 2(p-1) \} \{ DH_p(x) \} = 0, p \geqslant 1. \tag{17}$$

Thus both $H_{p-1}(x)$ and $DH_p(x)$, $p \geqslant 1$, are solutions of the same DE. Since both are polynomials (they have the same parity too), they cannot be the linearly independent solutions of the DE. Hence $DH_p(x) = C_p H_{p-1}(x)$, $p \geqslant 1$, C_p being the constant of proportionality. The familiar choice is $C_p = 2p$, for all $p \geqslant 1$, leading to the differential recurrence relation, reation (12).

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From (1), we have $\{D^2 - 2xD + 2(N+1)\} H_{n+1}(x) = 0$. Eliminating the derivatives using (12), we arrive at the three-term recurrence relation satisfied by the Hermite polynomials, relation (11).

Using (16), we have $H_{2m+1}(0) = 0$ One may be tempted to ask whether $H_{2m}(0)$ can also follow suit. We show that the answer is definitely in the negative. The *DE* for even degree polynomials is

$$(D^2 - xD + 4m) H_{2m}(x) = 0, \quad m \geqslant 0. \tag{18}$$

Since $H_{2m}(x)$ is an even polynomial in x (see (16), its derivative with respect to x is an odd polynomial in x and hence vanishes at $x = 0$. If the constant term of $H_{2m}(x)$ were zero, then the sotution of (18) and its derivative would be zero at the same point $x = 0$. Since an initial value problem in the theory of second order linear differential equations has a unique solution [14], the solution of (18) would be identically zero. As we are interested in having a non-trivial polynomial solution of the *DE*, we must choose $H_{2m}(0)$ to be non-zero.

Using (10), (12) and (16), we now show that $H_{2m}(0) \neq 0$ will also make $H_{2m+1}(x)$ a non-trivial polynomial. The Hermite polynomial of odd degree is an odd polynomial and hence is zero at the origin. The differential equation satisfied by it implies that it is not identically zero since its derivative is different from zero at the origin: $DH_{2m+1}(x)\mid_{x=0} = 2(2m+1) H_{2m}(0) \neq 0$.

It follows from the differential recurrence relation and (18) that

$$4(2m+2)(2m+1) H_{2m}(0) = D^2 H_{2m+2}(x)\mid_{x=0}$$
$$= -4(m+1) H_{2m+2}(0). \tag{19}$$

Hence $H_{2m+2}(0) \neq 0$, if $H_{2m}(0)$ is non-zero for a particular m. Iteration of (19) now yields that $H_{2m}(0) \neq 0$, for all $m \geqslant 0$. Since every non-zero $H_{2m}(0)$ saves both $H_{2m}(x)$ and $H_{2m+1}(x)$ from being identically zero, it is clear that a single non-zero $H_{2m}(0)$ can save

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the entire set of Hermite polynomials .

From the three-term recurrence relation, we have

$$H_{2m}(0) = -2\ (2m-1)\ H_{2m-2}(0), \quad m \geqslant 1. \tag{20}$$

Since every polynonial belonging to a set of simple polynomials has a precise degree with a non-zero leading term [4, 6], $H_0(0)$ cannot be zero if we demand Hermite polynomials to be simple polynomials. From (20) it follows that $H_{2m}(0) \neq 0,\ m \geqslant 0$.

If tne constant teim of $H_{2r}(x)$ were zero for some $r \geqslant 1$ then $H_{2r}(x) = x^2\ \{A_{2r}\, x^{2r-2} + \ldots + A_2\}$. (Here we use only the fact that $H_{2r}(x)$ has a definite parity and hence it contains only even powers of x.) Therefore, $H_{2r}(x)$ would have a zoro of multiplicity 2 at $x = 0$, contradicting the fact that the zeros of real orthogonal polynomials are distinct [4, 6]. Hence $H_{2r}(0) \neq 0$, for all $r \geqslant 0$.

What a tremendous potenial the particularly pleasant property $H_{2n}(0) \neq 0$ has! As $H_{2n}(0) \neq 0$, $H_n(x) \not\equiv 0$, $N \geqslant 0$. One member of a family saves the entire family. While it is good to know a better proof of a result, it is better to know good proofs of the same also. One may extend our approach to other real orthogonal polyuomials associated with an even weight function and an interval of orthogonality symmetric with respect to the origin (e. g. Legendre polynomsals and Chebyshev polyhomials [2, 3]).

4. Philosophy of the Hermite Polynomials

We now show that the Hermite polynomials can be our friends, philosophers, and guides [11] Blessed are those who are friendly with the Hermite polynomials [12]. The Hermite polynomials satisfy a simple but elegant and beautiful relation, relation (2). The meek are definitely blessed [15]. Blessed are those who are simple like the Hermite polynomials! Equations (2) and (5), and induction

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tell us that $H_n(x)$ has only *integers* as its coefficients. This property points out to us to be citizens of *integrity*. Blessed are those who maintain their integrity like the Hermite polynomials!

Next look at their Rodrigues' representaion. The Hermite polynomials silently tell us that they are neither delighted by the presence of $exP(+x^2)$ in their Rodrigues' representation nor dejected and depressed because $exp(-x^2)$ is also present in it. Calmly, every Hemite polynomial accepts both these two factors. Blessed are those who weigh in equal scale delight and dole [16, 17] like the Hemite polynomials!

Consider now their power series expansion. Terms with positive as well as negative coefficients are present, while the adjacent coefficients alternate in sign [7]. There are ups and downs in everyone's life. The Hermite polynomials gently remind us to maintain equanimity in all situations in life-whether it be prosperity or adversity [18]. Blessed are those who maintain their equilibrium under any circumstances!

$H_n(x)$ contains only those and all those powers of x which are congruent to n (*mod* 2) [7]; see also equation (7). All the powers of x which have the same nature as n (even or odd) are equal before $H_n(x)$. $H_n(x)$ is just impartial and magnanimous to *every* even (odd) power of x when n is even (odd). Blessed are those who accept and uphold the principle of equality like the Hemite polynomials! Blessed are those who are impartial and magnanimous like the Hermite polynomials!

All the terms of $H_n(x)$ unite beautifully and perfectly to give it a definite parity! Lesson: Our thoughts, words, and deeds should be in perfect unison (19). Happy are those who harmonize their thoughts, words, and deeds perfectly! Blessed are those who always put their creed into their deed and never speak with double tongue[19]!

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We learn from the three-term recurrence relation together with

$$H_0(x) = 1, \tag{21}$$

$$H_1(x) = 2x, \tag{22}$$

that the family of the Hermite polynomials is a well-knit family. Its members are so nicely united that if yon know *any* two of them, then you know all of them. Blessed are those who are friendly and united with the other members of their family and society!

Next let us have a look at the differential recurrence relation satisfied by the Hermite polynomials. However much we try, we cannot learn anything about the constant terms of the Hermite polynomials from the above relation. The differential recurrence relation doesn't let out its family secrets. Blessed are those who maintain secrecy about their family matters!

While three successive Hermite polynomials are connected by the three-term recurrence relation, the differential recurrence relation relation is a first order differential equation connecting two polynomials with adjacent indices. To have all indices alike, we have to pay a price [2]. The result is a second order linear differential equation, equation (10). Teaching: Be ready to pay a price for social justice; be ready to sacrifice. Peace comes to a society only when a spirit of sacrifice pervades the minds of its people [20]. Blessed are those who are ever willing to sacrifice in the interests of the society!

Next look closely at the orthonormal property. This relation reveals that the Hermite polynomials are linearly independent. Advice: Be indepepdent. Blessed are those who can stand on their own legs!

H for Hermite; *H* for humility [21] also. Humble $H_n(x)$ are, humble they have been, humble they shall ever be, to borrow from

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Dickens [22]. Blessed are those who are humble! Blessed are those who can generate a thing of beauty which is a joy for ever [23], like the generating function for the beautiful Hermite polynomilas!

The lessons we have drawn above are not unique: One has the freedom to draw lessons freely from the set in one's own way. In short [11],

H for Humility	*P* for politeness
E for Equality	*O* for Optimism
R for Reliability	*L* for Liderty
M for Magnanimity	*Y* for Yoga [24]
I for Impartiality	*N* for Nobility
T for Tenderness	*O* for Originality
E for Equanimity	*M* for Mercy
	I for Independence
	A for Authenticity
	L for Love
	S for Simplicity!

My dear fellow citizens of the Kingdom of Hermite Polynomials,

"Ask not what the Hermite polynonials can do for you;
Ask what you can do for the Hermite polynomials [25]!"

Acknowledgements

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FIGURE CAPTION

FIGURE 1: schematic diagram Showing the development of the various properties of the Hemite polynomials Full springs are shown inside circles. Squares enclose the associate starting points. Triangles stand for the supplementary conditions necessary to make the incomplete springs complete ones. We have not given the complete paths to arrive at all the properties, starting from a given spring. The following abbreviations have been used: (*a*) *AF*: Arfken's formula, (2) of text. (*b*) *BF*: Bell's formula, (6) of text. (*c*) *CAF*: Corollary to Arfken's formula, (5(of text. (*d*) *DE*: Differential equation. (*e*) DRR: Differntial recurrence relation. (*f*) *GF*: Generating function. (*g*) *LC*: Leading coefficient of each and every $H_n(x), n \geqslant 0$ $(=2^n)$; supplement to the

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differential equation. (*h*) *ONP*: Orthonormal property. (*i*) *OP*: Orthonormal property. (*A* knowledge of the leading coefficient or the normalization constant for every $H_n(x)$ makes it a complete spaing.) (*j*) *PRR*: Pure (three-term) recurrence relation. (*k*) *PSE*: Power series expansion. (1) *RR*: Rodigues' representation (*m*) *SD*: Supplement to the differential recurrence relation, (13) and (14) of text. (*n*) *SOS*: Spring of springs, (1) of text. (*o*) *SP*: Supplement to the pure recurrence relation, (21) and (22) of text.

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SOME CONSIDERATIONS BASED ON GRAVITATIONAL WAVES

By

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Human mind has forced to describe the physical properties of matter either as fields or as particles. Einstein used these concepts to formulate his General Theory of Relativity which is the most beautiful and profound creation. and still most logically complete and satisfying theory in existence. Also, the theory has two exciting predictions - the energy carrying gravitational waves and black holes. Black holes demand very strong gravitational fields - the gravitational potential should approach the square of the velocity of light for their formation, while gravitational waves exist even in the weak field approximation.

The distinctive feature of gravitational waves is their weak interaction with ordinary matter and with one another. Since the Universe came with a bang, one tempts to ask: does the universe consists primarily of gravitational waves? Can the density of gravitational waves (energy density divided by the square of the speed of light) at the present time exceed the density of ordinary matter, or atleast, the density of the electromagnetic background radiation? Finally, what is the gravitational wave spectrum?

It is difficult to give a direct experimental answer to these questions because of the weak interaction of scch waves with matter. However, we assess the role of gravitational waves which might fill the universe and helps us in investigating further in the deep realities of this universe.

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If we assume the waves are with wave length much shorter than the background radius of curvature, that is, if

$$| \nabla_{\alpha} \nabla_{\beta} h_{\mu\nu} | >> | R^{B}_{\alpha\mu\nu\beta} h_{\lambda\sigma} |$$

then they propogate on null geodesic and admit geometric optics. If the background is an isotropic universe, then it leads to

$$\mu'' + \mu \left[n^2 - \frac{a''}{a} \right] = 0 \qquad (1)$$

where $a = a(\eta)$ is the scale factor of the isotropic universe

$$ds^2 = a^2(\eta)(d\eta^2 - dx^2 - dy^2 - dz^2) \qquad (2)$$

and n is the priodicity of the wave.

This means that in the context of very early universe when the energy was of the order of $\sim 10^{19}$ Gev, there should have been a dackground radiation of gravitons like the background rediation of photons (microwave), and should have propogated with geometry, for instance,

$$R_{\mu\nu} \equiv \Gamma^{a}_{\mu a,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{a}_{\mu\nu} \Gamma^{\beta}_{\alpha\beta} + \Gamma^{\alpha}_{\mu\beta} \Gamma^{\beta}_{\nu a} = 0 \qquad (3)$$

in empty space. Empty space means that there is no matter present and no physical fields except gravitational field. The gravitational field does not disturb the emptyness whereas other fields do. This gives to a first order approximation.

$$\Gamma^{\alpha}_{\mu a,\nu} - \Gamma^{a}_{\mu\nu,\alpha} = 0 \qquad (4)$$

Making use of the coordinate condition (harmonic)

$$g^{\mu\nu} \Gamma^{\lambda}_{\mu\nu} = 0 \qquad (5)$$

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one finds

$$g^{\mu\nu} g_{\rho\sigma,\mu\nu} = 0 \tag{6}$$

That is, each $g_{\mu\nu}$ satisfies the d'Alembert equation, and its solution will consist of wave travelling with the velocity of light. They are gravitational waves or the geometry propogates. Unfortunately, it is not yet detected experimentally. Possibly, a reason could be seen as. In electrodynamics, the radiation is proportional to e^2, where e is the electronic charge. Here we must replace e^2 by Gm^2 where m is the mass of the electron and it should be considered at its lowest non-vanishing order (quadruple radiation). So,

$$\frac{Gm^2}{e^2} \sim 10^{-43} \tag{7}$$

It, therefore, follows that graviton emission will be atleast forty-three order smaller than photon emission.

Now we turn our attention to equation (1) above. This gives motivation to modify Einstein theory.

The gravity wave field

$$h_i^k = a^{-1} \mu(\eta) G_i^k \tag{8}$$

is interacting with the gravitational field of homogenious isotropic universe (2) has wave length

$$\lambda_0 = \frac{2\pi a}{n} \tag{9}$$

Physically, this equation is for an oscillator parametrically excited by a variable frequency. This equation can be regarded as the Schrödinger equation for a particle with energy n^2 in a potential $U(\eta) = \frac{a''}{a}$ with the η – time derivative playing the role of a spatial derivative

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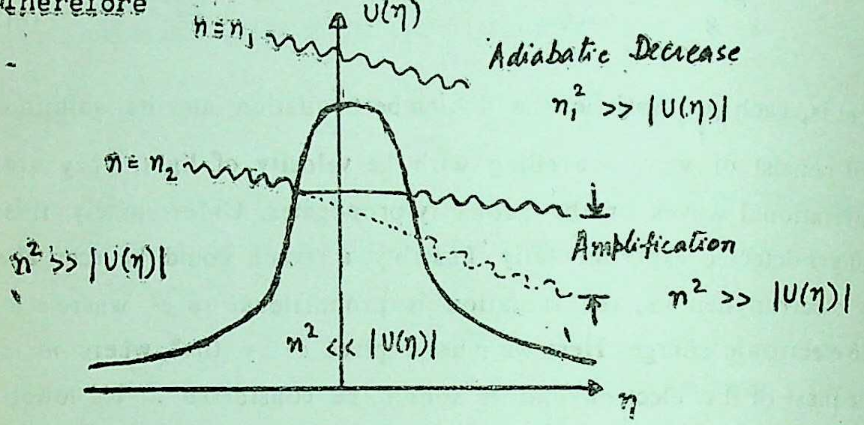
Therefore



In the domain where a wave with a given n satisfies the condition

$$n^2 >> |U(\eta)|$$

the amplitude of the wave decreases adiabatically, that is, according to law

$$h \sim e^{in\eta/a} \, .$$

However, in the domain where

$$n^2 << |U(\eta)|$$

the amplitude of the wave remains almost constant despite the growth of the scale factor $a(\eta)$. After penetrating the barrier and emerging in the domain where again

$$\eta^2 >> |U(\eta)|$$

the wave will have larger amplitude than it would have had if it had behaved according to the adiabatic law. This is the essence of the superadiabatic amplification of classical waves and quantum zero-point fluctuations.

Also one can see that the waves with different wave numbers n are amplified by different amounts. In this way an initial spectrum of the wave transform into final one.

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This means the gravition creation. So it helps us to understand classically the smoothening of initial inhomogenieties and anisotropies in the eraly universe.

In fact, all these are because equation (1) is not conformally invariant. It leads to a more fundamental question how to obtain a conformally invarient gravitational wave equations? Or, in turn, to modify Einstein equations of which gravitational waves are an inevitable consequence. This is one of the arguments to go for higher order theories of gravitation.

The Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R = X\, T_{\mu\nu} \tag{10}$$

are derivable from the Hilbert action

$$A = \int R\sqrt{(-g)}d^4x + x \sum_i \int m_i\, ds \tag{11}$$

Now, our problem goes to obtain a conformally invariant Hilbert action, possibly without introducing fields other than gravitation.

We demad this, because:

(i) replacing Hilbert Lagrangian (not conformally invarient) with the Lagrangian $g^{\mu\nu} \nabla_\mu \phi \ \nabla_\nu \phi + \xi R\phi^2$ (conformally invariant), which contains an extra masslesss scalar field leads to the consequences like "variation of gravitational constant G".

(ii) again, the suggestion by Weyl to use R^2 to make the field aetion scale invarient in order to unify gravitation with electromagnetism. It is conformally invariant theory, but it has a bad long range behaviour as it does not reduce to Newtonian theory in the linearised limit.

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(iii) another attractive alternatives suggested is $R^{3/2}$ so that the coupling constant in matter lagrangian is dimensionless. Also, R^n is used to study the behaviour of homogeneous isotropic universes or for the study of problem of gravitational instability in an expanding universe. Here n is a numerical constant. Also, various lagrangions are constructed on $R_{mv}\, R^{m}{}_{v}$, $R_{mnso}\, R^{mnso}$, R^2 and their combinations. But all these are not conformally invariant.

Therefore, the above does not meet our requirements. We, in fact, need some thing to nullify the manifestation of gravitation in the form of the term a''/a without taking recourse to special choice of the scale factor $a(\eta)$. So we propose the Lagrangian density in the form

$$L_g = \sqrt{(-g}\left[R - \sum_{m\neq 1}^{N} C_m \frac{(l^2R)^m}{\epsilon l^2} \right] \qquad (12)$$

where Σ stands for summation, m takes values 2,3,. . N, C_m are the arbitrary coefficients corresponding to m, 1 is the characteristics length such that tae charracteristics radii of curvature of the background would be large as compared with the gravitational wave length.

Unlike others we only one coupling constant (same as in the Hilbert case) that enters all the terms. The arbitrary coefficients C_m are dimensionless, and are introduced to cancell out the gravitational manifestation. In other words, C_m are to be determined so as to reduce zero the additional potential appearing in the gravitational wave equation (1) when we conformally transform Minkowski background without demanding restriction on the variation of $a(\eta)$. The arbitrary coefficients C_m may also be helpful in studying the trace anamolies in the stress tensor, and may also help in removing the ghosts in higher orider theores.

An application of variational principle to the action gives the equation of the modified theory as

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$$G_{\mu\nu} - \sum_{m\neq 1}^{n} \frac{mC_m}{6} (l^2R)^{m-1} \Big[R_{\mu\nu} - \frac{m}{2} g_{\mu\nu} R - (m-1)(R_{\mu;\nu} - g_{\mu\nu} \Box R)/R$$

$$- (m-2)(m-1) \left(R_{;\mu} R_{;\nu} - g_{\mu\nu} R_{;\alpha} R^{;\alpha} \right)/R^2 \Big]$$

$$= x\, T_{\mu\nu}$$

Here $T_{\mu\nu}$ stands for the energy momentum tensor responsible for the production of the gravitational potentianl $g_{\mu\nu}$, and satisfies

$$T^{\;\nu}_{\mu;\nu} = 0$$

as in General Relativity.

It is evident that the modified field equations contain additional terms involving $\Sigma \frac{mC_m}{6}(l^2R)^{m-1} \left[\quad \right]$. This is due to the correction introduced in the Hilbert Lagrangian R The Lagrangian (12) contain no field other than gravitation, and hence the field equation (13) is a natural modification of Einstein theory of gravi- tation. However, the equations (13) and (14) may be used to study the cosmological problem. The Lagrangian (12) contain a polynomial in R of a finite number of terms. This could not be disturbing because it is an observational fact that our universe is asymptotically flat. There is enough matter on our past light cone to cause it to refocuss. The total energy of the universe is exactly zero, the positive energy of the gravitons and matter particles being exactly compensated by the negative gravitational potential energy. That is why the universe is expanding. Also the unitarity is not well defined, except in scattering calculations in asymptotically flat spaces. Moreover, these terms may be able to find a higher-dimansional theory to cancell divergencies or to lead to a renormalizable theory.

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BIOMECHANICS OF CANCER

By

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Biomechanics of human body has become a subject of latest interest for the life scientists, engineers and the mathematicians. The study of this area has provided a better scientific knowledge of the human physiological system. The problems pertaining to blood flow, heat flow, diffusion through memberanes and tissues, kidney transport and pharmekokinetics have gained lot of attention of the scientists due to its dierect relevance with the everchanging science of medicine. The topics of bio fluid mechanics and bio diffusion have already been established as well known discipline.

Apart from the mechanics of normal physiological situations of a human body, focus has also been put on certain abnormal activities- Cancer is one of them. There are various types of cancer found and studied by the medical scientists.

Here we shtll discuss only malignant growth in human dermal regions and Polycythemia Vera (Blood Cancer). First we consider the existence of a malignant tumor in the underlying tissues of the epidermis of a human normal physiological functions, namely; self controlled metabolic activity, variable blood flow and perspiration. In the malignant portions the metabolic activity is continuous and uncontrolled. In this case we shall employ a mathematical model which has already been used extensively for normal regions, by Saxena [1,2], Saxena and Arya [3,4] and Saxena and Bindra [5,6].

The governing differential equation used by them is as follows

$$\frac{d}{dx}\left(K^{(j)} \frac{dT_i}{dx} \right) + M_j(x)\,(T_b - T_j) + S(x, T_j) = \rho\bar{c}\,\frac{\partial T}{\partial f} \qquad (1)$$

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v ı r

$K =$ thermal conducivity of tissue, $H = m_b C_b =$ blood mass flow rate, $S =$ metabolic heat generation, $T_b =$ body core temperature, $T =$ tissue temperature. Here the suffix 'j' is associated with the 'j^{th}' biological region under consideration.

Here we consider tumor situated in the dermis of the body (Fig. 1). In this case S is to be replaced by $\bar{S} + W$ where $\bar{S}$ is self controlled metabolic heat generation while W is uncontrolled metabolic heat generation. The main object of the analysis in this case is to find the abnormality in the temperature distribution due to this tumor. As it is already described earlier, this abnormality is due to change in blood perfusion, uncontrolled metabolism and rapid growth leading to the necrosis of the core. Myself and my coworkers have used various approaches to obtain the temperature profile in the tumor and the surrounding tissues with the help of analytical and numerical methods including variational finite element technique. This technique is most efficient and convencing. To make use of this the whole region is divided into smaller elements with each element can be prescribed different values to the physical and physiological parameters.

For a one dimensional cartesion symmetry, the mathematical descriptions for different cases are as follows.

CARE I:

(1) Epidermis (for $j = 1$)

$$K^{(1)} \quad K_1 \text{ (constant)}, \quad M_1=0, \quad \bar{S}_1=0, \quad W_1=0 \tag{2}$$

(2) Dermis (for $j = 2$ (1;4)

$$K^{(j)} = \lambda_j k_j + \beta_j \bar{K}$$

$$M_j(x) = \lambda_j m \left[\frac{x-a_1}{a_4-a_1} \right] + \beta_j \eta m \tag{3}$$

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$$\bar{S}_j(x,T_j) = \lambda_j \ s \left[\frac{x-a_1}{a_4-a_1} \right] \left[1+q_j(T_b-T_j) \right]$$

$$W_j = \beta_j \ \mu s \ , \ q_j = \frac{2}{T_b+T_a}$$

(3) Subcutaneous Tissues : ($j=5$)

$$K^{(5)} = k_5 \ \text{(constant)} , \ M_5 = m,$$

$$\bar{S}_5 = s\left[1+g_5 \ (T_b-T_5) \right], \ W_5=0 \ , \ q_5 = \frac{1}{T_b} \tag{4}$$

At the interfaces ($x=a_j$, $j=1,2,3,4$) following conditions are imposed:

$$T_j \ (x) = T_{j+1} \ (x) \tag{5}$$

and

$$K^{(j)} \frac{dT_j}{dx} = K^{(j+1)} \frac{dT_{j+1}}{dx} \tag{6}$$

SOLUTION:

The solution to *eq.* (1) in ech layer are given by

$$V_j = C_j f_j \ (y) + D_j g_j (y) + F_j (y) \tag{7}$$

for $j=1$ (1) 5.

where

$$\left.\begin{aligned} V_j &= \frac{T_b-T_j}{T_b} \\ \xi_j &= \frac{a_j}{\alpha_1} \end{aligned}\right\} \quad \text{for } j=1 \ (1) \ 5$$

$$y = \frac{x}{\alpha_1}, \ \alpha_1 = a_4 - a_1 \ , \ f_1 (y) = y, \ g_1 (y) = 1$$

$$F_1 (y) = 0$$

$$\left.\begin{aligned} f_j (y) &= \lambda_j \ A_i \ (z_{j-1}) + \beta_j \ e^{\sqrt{b_{j2}}\, y} \\ g_j (y) &= \lambda_j \ \beta_i \ (z_{j-1}) + \beta_j \ e^{-\sqrt{b_{j2}}\, y} \\ F_j (y) &= - \lambda_j \ N_{j3} - \beta_j \ N_{j4} \end{aligned}\right\} \quad j=2 \ (1)4$$

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$A_i(z_{j-1})$ and $B_i(z_{j-1})$ are Aioy's functions.

$Z_{j-1} = (b_{j1})(y-\xi_1)$, $j=2\,(1)\,4$

$$f_5(y) = e^{\sqrt{b_{51}}\,y}, \quad g_5(y) = e^{-\sqrt{b_{51}}\,y}$$

$$F_5(y) = -N_{52} \text{ (constant).}$$

CASE II.

The equation (1) alongwith boundary conditions

$$K \frac{dT}{dx} = h(T-T_a) = LE$$

and

$$T = T_b$$

is transformed into the following equivalent variational form

$$I_i = \tfrac{1}{2}\int_{a_{i-1}}^{a_i} \left[K^{(i)} \left(\frac{dT^{(i)}}{dx}\right)^2 + (M_b C_b)^i \left(T_A - T^{(i)}\right)^2 \right.$$

$$\left. - 2\left(\bar{S}^{(i)} + W^{(i)}\right) T^{(i)} \right\} dx + \tfrac{1}{2}\left[h\,(T - T_a)^2 + 2\,L\,E\,T^{(i)} \right]_{x=a_0}$$

$$\text{for } i=1\,(1)\,9 \qquad (10)$$

The elementwise assumptions are as given below:

$$K^{(i)} = K_i \text{ (constant) for } i = 1(1)\,9$$

$$(M_b C_b)^{(i)} = 0 \qquad \text{for } i = 1,2$$

$$(m_b C_b)^{(i)} = \alpha_i\, m \frac{(x-a_0)}{a_7 - a_2} + \beta_i m, \; i=3\,(1)\,7$$

$$(m_b C_b)^{(i)} = m \text{ (constant)}, \qquad i=8,9$$

$$\bar{S}^{(i)} = 0, \quad W^{(i)} = 0, \qquad i = 1,2$$

$$\bar{S}^{(i)} = \lambda_i\, s \left[\frac{(x-a_1)}{(a_7 - a_1)}\right]\left[1 + q_i\,(T_i - T_{i-1})\right], \; i = 2,7$$

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$$\bar{S}^{(i)} = \lambda_i s \left[1 + q_i (T_i - T_{i-1}) \right], \qquad i = 8,9$$

$$W^{(i)} = W_i \text{ (constant)}, \qquad i = 3\,(1)\,9$$

$$q_i = \frac{2}{T_b + T_a} \qquad i = 2\,(1)\,7$$

$$q_i = \frac{1}{T_b} \qquad i = 8, 9$$

SOLUTION:

The following linear variation with respect to x is assigned to $T^{(i)}$ in each element

$$T^{(i)} = A_i + B_i x \tag{11}$$

where
$$\left.\begin{aligned} A_i &= \frac{(a_i T_{i-1} - a_{i-1} T_i)}{(a_i - a_{i-1})} \\ B_i &= \frac{(T_i - T_{i-1})}{a_i - a_{i-1})} \end{aligned}\right\} \qquad i = 1\,(1)\,9$$

The integral for whole SST region is given by

$$I = \sum_{i=1}^{9} I_i \tag{12}$$

where I_i $(i=1(1)9)$ are the integrals for each element.

I is extremized $w.r.t.$ T_i for $i = 0(1)8$ and the following system of algebraic equations is obtained

$$\bar{A}\, T = \overline{B} \tag{13}$$

where

$$\left.\begin{aligned} T &= [\, T_0, T_1\, T_2, \ldots, T_8 \,] \\ \text{and} \quad \overline{B}\,' &= [\, \overline{B}_0, \overline{B}_1, \ldots \overline{B}_8 \,] \end{aligned}\right\} \tag{14}$$

Here T' and $\overline{B}\,'$ are transpose of vectors T and $\overline{B}$.

The system of *eqs.* (13) is solved to obtain nodal temperatures which in turn are substituted in $T^{(i)}$ to give temperature profile in each element.

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CASE III

The partial differential equation for heat flow in skin and subcutaneous tissues for a two dimensional steady state may be given as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\ m_b\ C_b\,(T_p-T)+S=0 \tag{15}$$

where

$$S=\bar{S}+W \tag{16}$$

We consider a rectangular composite region with boundaries $x=0$, $x=48a$, $y=0$ and $y=20a$ with the following conditions:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=\xi \tag{17}$$

$$-\left.\frac{\partial T}{\partial x}\right|_{x=48a}=\bar{\theta} \tag{18}$$

$$T\,(x,0)=T_b \tag{19}$$

$$-\ K\frac{\partial T}{\partial y}=h\ (T-T_a)+L\ E \qquad \text{at } y=20a \tag{20}$$

The variational formulation of (15) alongwith the boundary conditions may be given by

$$I^{(e)}=\tfrac{1}{2}\int\int_{\Delta^{(e)}}\Big[\ K^{(e)}\Big(\frac{\partial T^{(e)}}{\partial x}\Big)^2+K^{(e)}\Big(\frac{\partial T^{(e)}}{\partial y}\Big)^2+$$

$$(m_b\,C_b)^{(e)}\,(T_b-T^{(e)})^2-2\,(\,\bar{S}+W^{(e)})\,T^{(e)}\Big]\,dx\,dy$$

$$+\ \delta^{(e)}+\sigma^{(e)}+\Omega^{(e)} \tag{21}$$

where $\Delta^{(e)}$ is the region contained in the eth element and

$$\delta^{(e)}=\tfrac{1}{2}\int_{T_1^{(e)}}\Big[\ h\ (T^{(e)}-T_a)^2+2\,L\,E\,T^{(e)}\Big]\,dx$$

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$$\sigma^{(e)} = \int_{T_2^{(e)}} \xi^{(e)} \, T^{(e)} \, dy$$

$$\Omega^{(e)} = \int_{T_3^{(e)}} \theta^{(e)} \, T^{(e)} \, dy$$

CASE IV.

Now, we consider the existence of "POLYCYTHENIA VEHA" in a human body. This disease causes gradual change in the blood viscosity resulting reduction in the supply to the peripheral parts having dense capillary network. Consequently the adjoining body cells receive less nutrition and food which effects the metabolic heat generation. Thus the heat balance in the human body is disturbed and the temperature profiles change with time.

Approximating the change in physilogical processes following functions have been substiuted for the parameters involved in

the equation (1)

$$\begin{aligned} K &= k \\ M &= m\,a\, e^{-\beta t} \\ S &= s \sqrt{\;} e^{-\delta t} \;(T_b - T) \end{aligned} \qquad (22)$$

Eq. (1) and Eq. (8) are solved using (22) to give

Outer Skin Surfaee

$$\frac{\partial v}{\partial y} = \frac{lh}{k} V - \left[\frac{lh}{k} V_a + \frac{LE\,l}{kT_b} \right] af\; y = 0 \qquad (23)$$

SST region

$$\frac{\partial^2 v}{\partial y^2} - \left[\frac{\partial^2}{k} (m a e^{-\beta t} + S \sqrt{\;} e^{-\delta t}) \right] V = \left(\frac{l^2 \varrho \bar{c}}{k} \right) \left(\frac{\partial v}{\partial t} \right)$$

$$\text{for } \; 0 \leqslant v \leqslant 1 \qquad (24)$$

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Inner boundary

$$V = 0 \quad at \; y = 1 \tag{25}$$

Initial condition

$$V(x, 0) = 0 \tag{26}$$

as $\quad T = T_b \qquad at \; t = 0$

Applying Laplace transform on (23) — (25) and using (26), we have

$$\frac{d\bar{V}}{dy} = n_1 \bar{V} - n_2 \qquad at \; y = 0 \tag{27}$$

$$\frac{d^2\bar{V}}{dy^2} - (A_2 + A_3 p)\,\bar{V} = A_4 \sum_{i=1}^{m} \frac{a_i}{p^i} + A_5 \sum_{i=1}^{n} \frac{b_i}{p^i} \tag{28}$$

$$\text{for} \qquad 0 \leqslant y \leqslant 1$$

and $\quad \bar{V} = 0 \qquad \text{at } y = 1 \qquad (29)$

SOLUTION :

The solution for the *SST* region is obtained in the following form:

$$V = A_{16}\,[1 - exp(-A_{15}\,t)]\,(y - 1) + A_{17}\,(y - 1)$$

A comparative study of the change in temperature profiles between normal conditions and malingnency have been shown in Figs [3, 4, 5].

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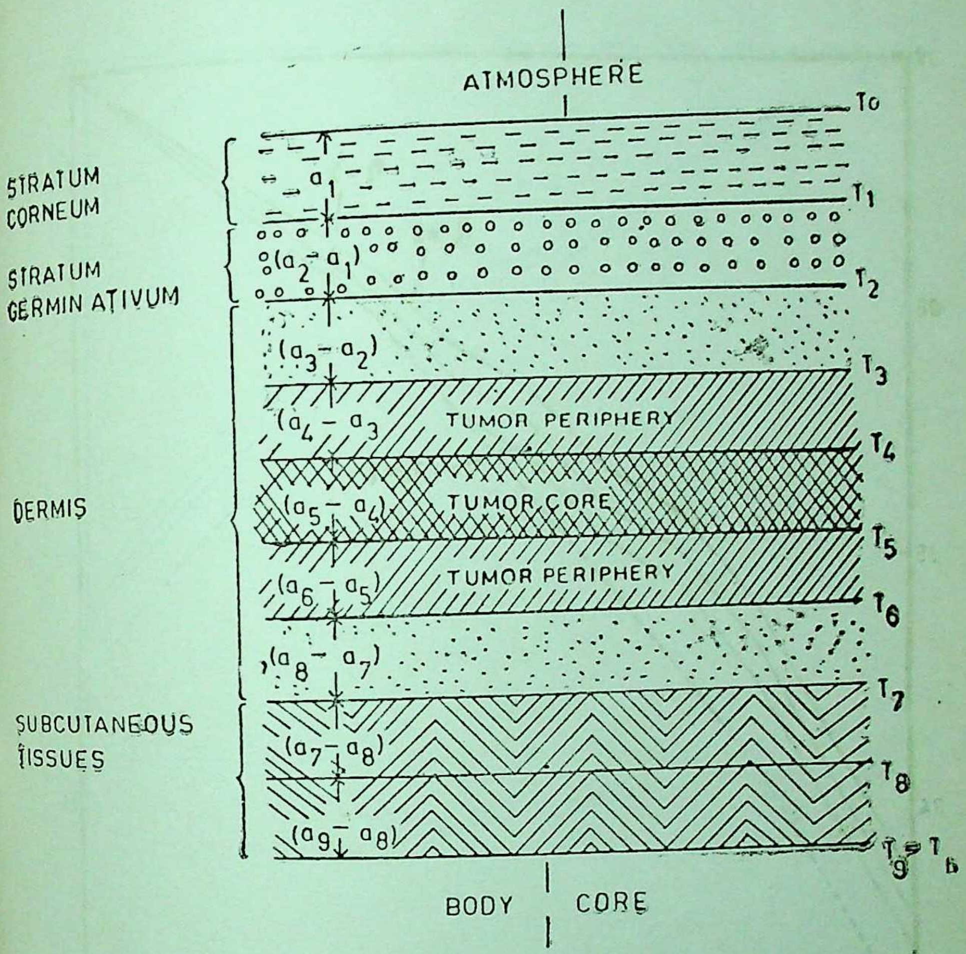

FIG.1 ONE DIMENSIONAL FINITE ELEMENT MODEL OF SKIN AND SUBCUTANEOUS TISSUES WITH A TUMOR IN DERMIS.

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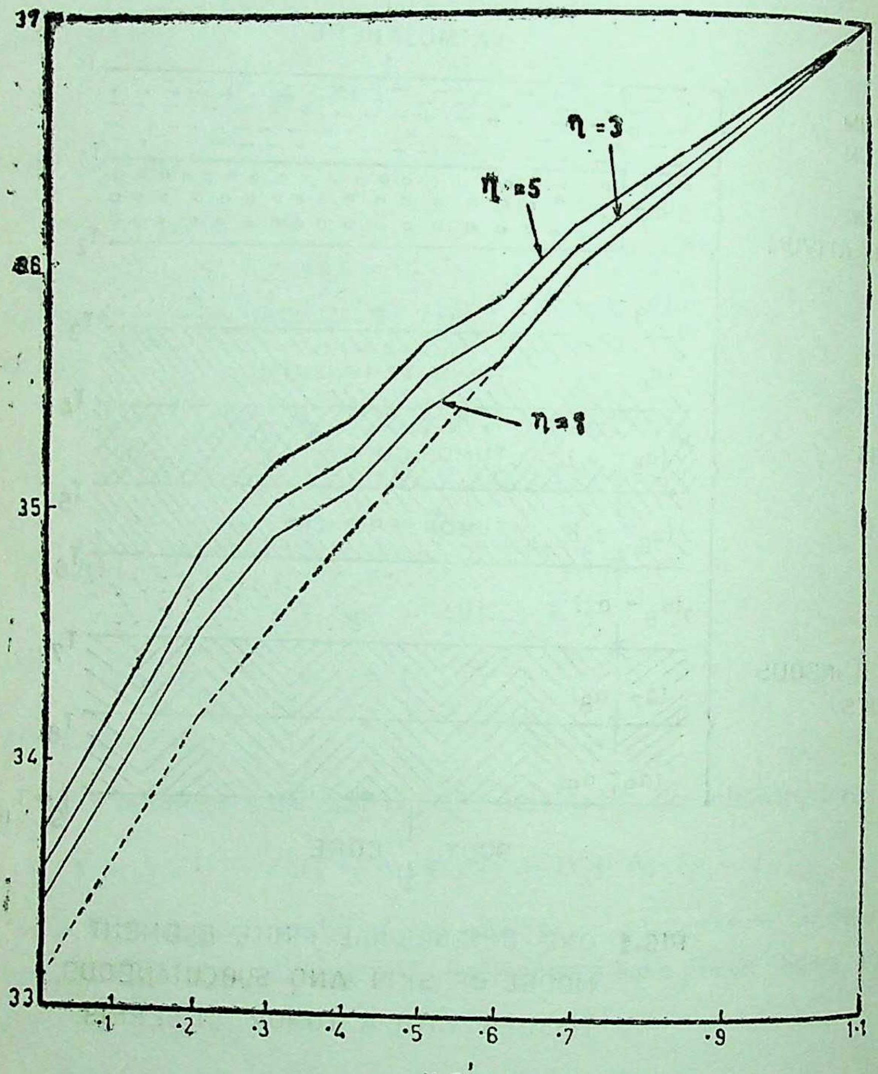

FIG. 3. GRAPH BETWEEN TEMPERATURE T AND POSITION X FOR $T_a = 15\,^{\circ}C$ AND E = 0. BROKEN LINE IS FOR BENIGN TISSUES.

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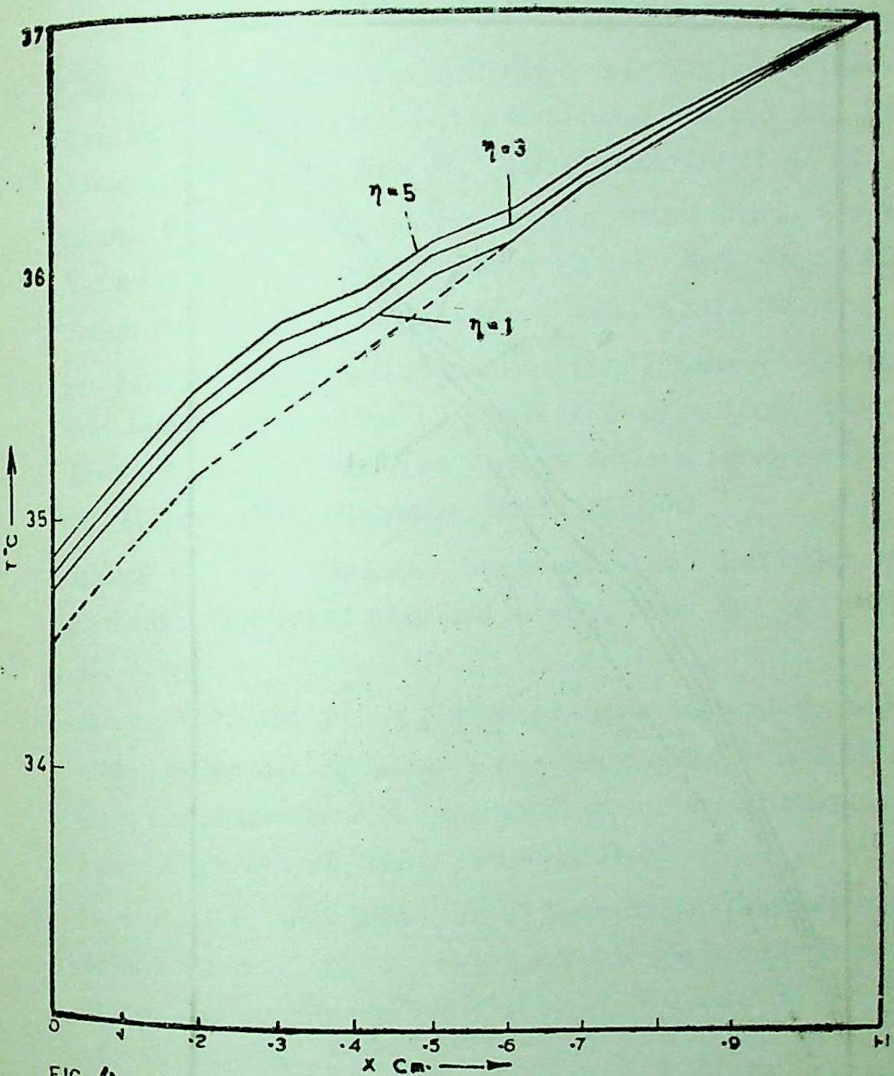

FIG. 4 GRAPH BETWEEN TEMPERATURE T AND POSITION X FOR $T_a = 23°C$ AND $E = 0$. BROKEN LINE IS FOR BENIGN TISSUES.

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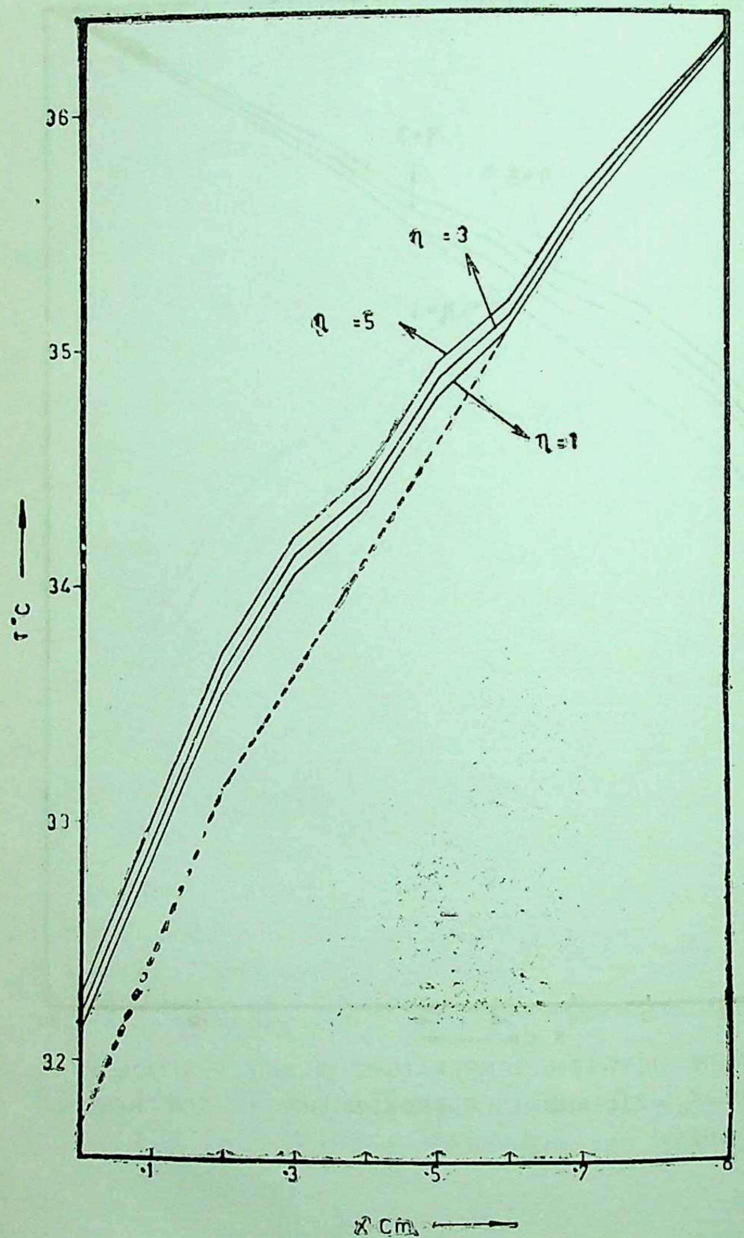

FIG. 5 GRAPH BETWEEN TEMPERATURE T AND POSITION X FOR $T_a = 23°C$ AND $E = 0.24 \times 10^{-3}$ gm/cm^2-min BROKEN LINE IS FOR BENIGN TISSUES.

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Proc VPI., Vol. 2, 1990

SOME NORMALITY LIKE CONDITIONS IN TOPOLOGICAL SPACES

By

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1. Introduction. Tychonoff spaces are perhaps the most natural generalisation of metric spaces in the sense that these are precisely the spaces which are associated with uniformities where the metric space notion of completeness has its most general setting. These are also the spaces which have Hausdorff compactification and are embeddable in cubes. However, a slightly stronger condition has been found more powerful in the development of a unifying theory. This is the notion o. normality, where any pair of disjoint closed sets can be separated by diszoint open sets. Among its significant properties are

(a) A rich supply of continuous functions a consequence of Urysohn's Lemma)

(b) *C* and *C** - embeddability of every closed subset (Tietzei's Theorem) ;

(c) Insertibility of continuous functions between semicontinuous functions (Katetov's Theorem) ; and

(d) The normality of every locally finite open cover (a consequence of Stone's Theorem).

It is because of these important properties that normal spaces have been studied in literature in great details and in many directions. One such direction of work has been the study of sone weaker and some stronger conditions than normality. Below we describe some such work.

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2. Conditions weaker than normality.

A subset of a topological space is said to be *regular closed* if it is the closure of an open set A finite intersection of regular closed sets is called a π - *closed set*. A set which is both closed as well as open is called a *clopen set*. Clearly for a set clopen $\Rightarrow$ regular closed $\Rightarrow$ π closed. Using these special type of closed sets in the definition of normality, several weaker forms have been studied

Singal and Arya [6] defined a space to be *almost normal* if every pair of disjoint closed sets, one of which is regular closed, are contained in disjoint open sets. Singal and singal [7] defined a space to be *mildly normal* if every pair of disjoint regular closed sets are separated by disjoint open sets. Recently Lal and Rehman [3] studied *quasi normal* spaces where disjoint π closed sets are separated by disjoint open sets. It is clear that

Normal $\Rightarrow$ almost normal $\Rightarrow$ mildly normal, and

Normal $\Rightarrow$ quasi-normal $\Rightarrow$ mildly normal.

None of these implications is reversible in general. It may be mentioned have that regular mildly normal spaces were earlier studied by Shchepin [5] and regular quasi-normal spaces were studied by Zaitsev [8].

Mack [4] defines a space to be *m normal*, *m* being an infinite cardinal if a pair of a closed set and a regular *Gm* set are separated by disjoint open sets. Here a set is clefined to be a *regular Gm* set if it is intersection of almost *m* closed sets whose interior contains the set. If *m* is countable then the space is called δ*-normal*. Clearly every normal space is *m* normal and a compact T_1, non T_2 space is *m* normal, but not normal.

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Mack [4] also defincs a space to be δ *normaally separated* if each closed set and each zero set disjoint from it are completely separated. A space is defined to be *weakly* δ-*normally scparated* if each regular closed set and a zero set disjoint from it are completely saparated. It is clear that

Normal ⇒ δ normally separated ⇒ weakly δ normally separated. It is known that none of these implications is reversible.

3. *Stronger forms* of *normality*.

Ellis [1] defined a sgace to be *ultra normal* if disjoint closed sets are contained in disjoint clopen sets. His main aim was to study extension properties of such spaces. In this directon he has provcd, among other thnigs, the following:

(a) Let A be a closed subset of an ultranormal sqace X, and lct Y be a complcte separable metric space. Then every (bounded) continuous function $f: A \to Y$ has a (bounded) continuous extension $F: X \to Y$.

(b) Let d be a bounded continuous pseudo-ultrametric on a closed subset A of an ultranormal space X such that (A, d) is separable. Then there is a bounded D on X which extends d such that $\mathrm{diam}(X) = \mathrm{diam}(A)$.

Here a pseudo-ultrametric on a set X is a pseudo metric d which salisfies the strong inequality

$$d(x, y) \leqslant \max\{d(x, z), d(z, y)\}.$$

We studied these spaces recently [2] and some of the results proved are as follows:

(c) A topological space X is ultra-normal iff for each pair of disjoint closed subsets A and B of X, there exists a $I_{clo,0}$ map $f: X \to [0, 1]$ such that $f(A) = 0$, $f(B) = 1$.
Here $I_{clo,0}$ map means a map under which inverse image of each set is a clopen set.

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(d) If every $I_{clo,o}$ map f from a closed subset of X can be extended to a $I_{clo,0}$ map F on X, then X is ultranormal.

(e) If A is a closed subset of an ultranormal space X, then every continuous map $f: A \to [0,1]$ can be extended to a $L_{clo,0}$ map F on X.

Here $f: X \to Y$ is defined to be $L_{clo,o}$ at $x \in X$, if for each open set U containing $f(x)$, there exists a clopen subset V of X such that $f(V) \subset U$. f is said to be $L_{clo,0}$ if it is $L_{clo,0}$ at every $x \in X$.

We also introduce [2] two stronger forms of ultra normality. We say that X is *hereditarily ultranormal,* if every subspace of X is ultra-normal. X is said to be *completely ultranormal*, if any two separated sets in X are contained in disjoint clopen sets. It is shown that

(f). For a space X, completely ultra normal $\Rightarrow$ hereditarily ultra-normal $\Rightarrow$ ultranormal. None of the implications is reversible.

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A TALK ON
NONLINEAR WAVE PROPAGATION IN ARTERIES

By

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Before reporting the recent advances in nonlinear wave Fropagation in arteries, it is worth while to mention the following:

Practical aims of fluid Dynamics

The following are two aims of fluid Dynamics namely,

1) Improved locomotion through fluid media.

2) Enhanced understanding of transport by fluids in circulating systems.

This led to the following :

i) The design of afficient vehicles for locomotion in water or through the air, and

ii) better analysis of how great circulating systems (the atmosphere, the hydrologic cycle, or fluid flow systems in industrial plant) transport heat or various substances in solution or suspension. We do not concentrate on external fluid dynamics here and while discussing *internal fluid dynamics* the complications of human boody are as given below

1. Unusual range of Reynolds number
2. Unusual multiplicity of tube branching
3. Unusual distensibility of containing vessels
4. Unusual pulsatility
5. Unusual fluid properties

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Before comming to the topic it will be necessary to make it clear that there are three functions of blood flow namely,

i) Carrying (ii) Distribution (iii) Diffusion

The present talk is based on the investigations of non-linear wave propagation phenomena in arterial vessels embedded in the tissues. In addition to our work the works of following are worth mentioning: Mc donald [1] Funy [2], Skalak [3], Morgan-Kiely [4], Womersley [5], Jager et. al [6], Mirsky [7], Atabek [8], Lamhert [9], Anliker et. al. [10], Ling and Atabek [11].

A theoretical model of non linear wave propagation in arteries by including the non-linear motions of both the blood vessels and the surrounding tissue is a point of interest.

A complete model of pules wave includes the three components (i) the flow of blood driven by the presence gradient under the control of vessel lumen elasticlty; (ii) the motion of vessel wall with surrounding tissues caused due to the action of the blood flow on the wall; (iii) the boundary conditions between the blood and wall in contact. No doubt, the flow of blood can expressed by means of the idcompressible, viscous flow equations but the motion of arterial wall is quite important and needs constitutive equations to describe the motion of an arterial wall.

The following four assumptions have been made:

(1) The blood vessels are locally triclinic, anisotropic elastic and incompressible and axisymmetric bodies of revolution. The vessel walls are thin as compared with the overall cross-sectional dimension.

(2) The forces acting on the blood vessels will not affect the local orthoisotropy of the vessels.

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(3) The surrounding tissue of the blood vessel is modelled as a thin elastic medium for the aorta or as an elastic medium with the finite thickness for the arteries connected with peripheral tissues, muscle and other substances, the stress wave propngation in such a medium is dispersed with the increase of the radial distance from the vessel wall.

(4) Gravitational forces are neglected.

The fundamental equations by Wu and Lee [12] have been taken in base for the equations of model.

Velocities and Accelarations of Arterial Walls.

Lat v_r and v_r be the axial and radial displacements of a point on the arterial wall, then

$$v_r = R_w - R^* \tag{1}$$

R^* being the radius of the midsurface in equilibrium or at undeformed state.

For an axisymmetrical motion, the velocity components of the wall can be defined to be

$$q_z = \frac{\partial v_z}{\partial t} \quad ; \quad q_r = \frac{\partial v_r}{\partial t} \tag{2}$$

From the theory of continuum mechanics, the accelaration components of wall can be defined as

$$a_z^e = \frac{\partial q_z}{\partial t} + q_z \frac{\partial q_z}{\partial z} + q_r \frac{\partial q_z}{\partial r}$$

$$a_r^e = \frac{\partial q_r}{\partial t} + q_z \frac{\partial q_r}{\partial z} + q_r \frac{\partial q_r}{\partial r} \tag{3}$$

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Since the thickness of the wall is thin, the spatial variation of velocity components along the thickness of the wall is negligible, i. e. ($\frac{\partial q_z}{\partial r}$) and $(\partial q_r / \partial r)$ in equation (3) can be ignored. Thus equation reduces to

$$a_z^e = \frac{\partial^2 v_z}{\partial t^2} + \frac{\partial v_z}{\partial t} \frac{\partial}{\partial z} \left(\frac{\partial v_z}{\partial t} \right)$$

$$a_r^e = \frac{\partial^2 v_r}{\partial t^2} + \frac{\partial v_z}{\partial t} \frac{\partial}{\partial z} \left(\frac{\partial v_r}{\partial t} \right) \tag{4}$$

Equrtions of Motion of the vessel-surrounding Tissne system

An important thing in analysis of non-linear wave propagations of blood vessels is how to determine or eliminate the unknown constrained stresses of the peripheral tissue in the equations of motion. Some of the authors used Euler's relationship namely the blood pressure vs area (or radius) to get mathematical solutions to wave propagation problems while some others used an elastic vibration system consisting of inertia, mass and friction to model the effect of surrounding tissues But these approaches do not provide clear physical explanations of the effects of the motions of the blood vessels on the surrounding tissue on the wave propagation behaviour.

Here it is assumed that the peripheral tissue around a blood vessel is an axisymmetric elastic medium and that the wave propagation in such a medium will be dispersed with the increase of radial distance.

Let ρ_t denote the density of the tissue, a_z^t and a_r^t the acceleration camponents of a material point in the tissue and σ_r, σ_θ and σ_z be the physical normal stresses in cylindrical coordinates (r, θ, z). Since the motion of the tissue is axisymmetric, in a cylindrical coordinate system there exists only one shear stress T_{rz}. The equations of motion of surrouding tissue can be written as:

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$$\rho_t \, a_z^t = \frac{\partial \sigma_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rT_{rz})$$

$$\rho_t \, a_r^t = \frac{\partial T_{rz}}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\sigma_r) - \frac{\sigma_\theta}{r} \tag{5}$$

This equation now can be reduced to a system of equations expressed in terms of the boundary variables of the blood vessel. Let R_o and R_1 denote the outer radii of the blood vessel and the peripheral tissue respectively. Then from (5), by integration between limits from R_0 to R_1, we have

$$\int_{R_0}^{R_1} \rho_t \, a_r^t \, r\,dr = (\sigma_r)_1 R_1 - (\sigma_r)_0 R_0 + \int_{R_0}^{R_1} \frac{\partial T_{rz}}{\partial z} r\,dr - \int_{R_0}^{R_1} \sigma_\theta . \, dr \tag{6}$$

$$\int_{R_0}^{R_1} \rho_t \, a_z^t \, r\,dr = (T_{rz})_1 R_1 - (T_{rz})_0 R_0 + \int_{R_0}^{R_1} \frac{\partial \sigma_z}{\partial z} r\,dr \tag{7}$$

where the subscripts 0 and 1 indicate the valus of the variable on the outer boundari of the blood vessel and the peripheral tissue.

Since

$$\frac{\partial}{\partial z} \int_{g(z)}^{h(z)} f(z, r)\,dr = \int_{g(z)}^{h(z)} \frac{\partial}{\partial z} f(z,r)\,dr + f(z, h)\,h'(z) - f(z, g)\,g'(z)$$

equations (6) and (7) can be expressed as:

$$\int_{R_0}^{R_1} \rho_t \, a_r^t \, r\,dr = -p_a R_1 + p_t R_0 + T_t R_0 \frac{\partial R_0}{\partial z} + \frac{\partial}{\partial z}\int_{R_1}^{R_0} T_{rz}\,rdr - \int_{R_0}^{R_1} \sigma_\theta \, dr \tag{8}$$

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$$\int_{R_0}^{R_1} \rho_t \, a_z^t \, r dr = p_a \, R_1 \, \frac{\partial R_1}{\partial z} + T_t R_0 - p_t R_0 \, \frac{\partial R_0}{\partial z} + \frac{\partial}{\partial z} \int_{R_0}^{R_1} \sigma_z \, r dr \quad (9)$$

Let us put

$$a_z^t = (a_z^e)_0 \, F_1 \, (r/R_0) \, , \quad a_r^t = (a_r^e)_0 \, F_2 \, (r/R_0);$$

$$\sigma_z = (\sigma_z)_0 \, F_3 \, (r/R_0) \quad , \quad \sigma_\theta = (\sigma_\theta)_0 \, (r/R_0) \, F_4 \, (r/R_0) \quad ,$$

$$T_{rz} = (T_{rz})_0 \, F_5 \, (r/R_0) \quad (10)$$

Then keeping in view the stress relationships on the outer and inner boundaries of the surrounding tissue, equations (8) and (9) reduce to

$$\rho_t \alpha_2 R_0 \, a_r^e = - p_a \frac{R_1}{R_0} + p_t + T_t \, \frac{\partial R_w}{\partial z} + \frac{1}{R_0} \frac{\partial}{\partial z} \left[\frac{\alpha_5 (S_\xi + p_w) R_0^2}{1 + (\partial R_w / \partial z)^2} \frac{\partial R_w}{\partial z} \right]$$

$$- \alpha_4 \, S_\theta \quad (11)$$

$$\rho_t \alpha_1 R_0 \, a_z^e = p_a \, \frac{R_1}{R_0} \frac{\partial R_1}{\partial z} - p_t \, \frac{\partial R_w}{\partial z} + T_t + \frac{1}{R_0} \frac{\partial}{\partial z} \left(\frac{\alpha_3 \, S_\xi \, H_0^2}{1 + \left(\frac{\partial R_w}{\partial z} \right)} \right) +$$

$$\frac{\alpha_3 \, R_0}{1 + (\partial R_w / \partial z)^2} \quad \frac{\partial}{\partial z} \, T_w \, \frac{\partial R_w}{\partial z} - P_w \left(\frac{\partial R_w}{\partial z} \right)^2) \quad (12)$$

where p is the circumstance pressure and

$$\alpha_1 = \frac{1}{{}^2R_0} \int_{R_0}^{R_1} F_1 \, (r/R_0) \, r dr = \int_1^{\lambda_1} F_1 \, (\lambda) \, \lambda \, d\lambda \, ; \quad (i=1, \ldots, 5; \; \lambda = r/R_0)$$

--- (13)

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Equations (11) and (12) are the basic system of equations for the motion of surrounding tissues expressed in terms of the variables of the vessel walls. These equations hold for both the small tissue thickness and the finite tissue thickness.

Now, deduce equations of motion for the vessel tissue system by using equations of motion surrounding tissue. Equations of motion of blood vessels (Wu and Lee) are

$$\rho_e\, a_r^e = \frac{1}{h\,[1+(\partial R_w/\partial z)^2]^{1/2}} \Bigg[p_w - p_t + (T_w - T_t\, \frac{\partial R_w}{\partial z} + \frac{h}{R_w} \Big(\frac{S_\xi\, R_w}{1+(\partial R_w/\partial z)^2]^{1/2}} \frac{\partial R_w}{\partial z} \Big) \Bigg] - S_\theta / R_w \quad \ldots (13)$$

and

$$\rho_e\, a_z^e = \frac{1}{h\,[1+(\partial R_w/\partial z)^2]^{1/2}} \Bigg[(p_t - P_w)\, \frac{\partial R_w}{\partial z} + T_w + \frac{h}{R_w} \frac{\partial}{\partial z} \Big(\frac{S_\xi\, R_w}{[1+(\partial R_w/\partial z)^2]^{1/2}} \Big) \Bigg] \quad \ldots (14)$$

where h denotes the thickness of the vessel wall and is a function of z and t. Now eliminating unknown stresses of the peripheral tissue by substituting equations of motion of the surrounding tissue in (13) and (14) we have the equations of the vessel wall-tissue system as

$$\frac{\partial^2 u_r}{\partial t^2} + \frac{\partial u_z}{\partial t} \frac{\partial^2 u_r}{\partial z \partial t} = \frac{1}{\phi_1} \Bigg[-p_a \frac{R_1}{R_0} + p_w + T_w \frac{\partial R_w}{\partial z} - S_\theta \,(a_4 + \frac{h}{R_w} [1+(\partial R_w/\partial z)^2]^{1/2} + \frac{h}{R_w} \frac{\partial}{\partial z} \Big(\frac{S_\xi\, R_w}{[1+(\partial R_w/\partial z)^2]^{1/2}} \frac{\partial R_w}{\partial z} + \frac{1}{R_0} \frac{\partial}{\partial z} \Big(\frac{a_5\,(S_\xi + p_w)\, R_0^2}{1+(\partial R_w/\partial z)^2} \Big) \frac{\partial R_w}{\partial z} \Big) \Bigg] \quad \ldots (15)$$

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$$\frac{\partial^2 v_z}{\partial t^2} + \frac{\partial v_z}{\partial t}\frac{\partial^2 v_z}{\partial z \partial t} = \frac{1}{\phi_2}\Big[p_a \frac{R_1}{R_0}\frac{\partial R_1}{\partial z} - p_w \frac{\partial R_w}{\partial z} + T_w$$

$$+ \frac{h}{R_w}\frac{\partial}{\partial z}\left(\frac{S_\xi R_w}{[1 + (\partial R_w/\partial z)^2]^{1/2}} \right) + \frac{1}{R_0}\frac{\partial}{\partial z}\left(\frac{a_3 S_\xi R^2{}_0}{1 + (\partial R_w/\partial z)^2} \right)$$

$$+ \frac{a_3 R_0}{1 + (\partial R_w/\partial z)^2}\frac{\partial}{\partial z}\left(T_w \left(\frac{\partial R_w}{\partial z} - p_w \left(\frac{\partial R_w}{\partial z} \right) \right) \right) \Big] \quad \ldots (16)$$

where

$$\phi_n = l_e \left[h \left(1 + (\partial R_w/\partial z)^2 \right]^{\frac{1}{2}} + R_0 \,\alpha_n\, (\rho_t/\rho_e) \, ; \qquad (n=1,2)$$

ϕ_1, ϕ_2 being the incorporation mass per unit area. In equations (15) and (16) α_i $(i=1, \ldots, 5)$ are the reduced coefficients of surrounding tissues, α_1, α_2 bing the axial and radial inertia coefficients respectively, a_3, a_4, and a_5 reduced coefficients of tissue stresses. These parameters may be determined by experiments on the non-linear wave propagations. However, for small tissue thickness, these parameters can be analytically expressed as

$$\alpha_i = [1 + (\partial R_w/\partial z)^2]^{1/2} \left[(h_t/R_0) + t_i (h_t/R_0)^2 + \delta_i (h_t/R_0)^3 \right]$$

$$(i=1, \ldots, 5) \qquad \ldots (17)$$

whee t_i, δ_i are the experimental constants. δ_1 can be neglected in comparison with t_i. If we put $t_i = \delta_i = 0$ and neglect the difference of mechanical properties between wall and tissue, then equations (15) and (16) will be the equations of motion of a whole body which consists of vessel wall and peripheral tissue.

Governing Equations of the blood flow

The blood is an incompressible fluid consisting of particulate phase of red cells, white platelets and erythrocytes. The flow of boold in general, can be described in terms of the momentum equations and the

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continuity equation of the incompressible viscous two phase flow. For an axisymmetric laminar flow, the governing equations can be expressed as:

Fluid Phase

Momentum Eqations

$$\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} \right) = \frac{\partial T_{rz}}{\partial z} + \frac{1}{r} \frac{\partial (r T_{rr})}{\partial r} - \frac{T_{\theta\theta}}{r} + \frac{KN}{\rho} (w_r - u_r) \tag{18}$$

$$\rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) = \frac{\partial T_{zz}}{\partial z} + \frac{1}{r} \frac{\partial (r T_{zr})}{\partial r} + \frac{KN}{\rho} (w_z - u_z) \tag{19}$$

Continnity Equation

$$\frac{\partial u_z}{\partial z} + \frac{1}{r} \frac{\partial (r u_r)}{\partial r} = 0 \tag{20}$$

where ρ is the density of the blood; u_r, u_z denote the components of velocity of blood flow along the r and z directions respectively, T is the stress tensor of blood, T_{rr}, $T_{\theta\theta}$ and T_{zz} represent the three normal stress components of T and T_{rz} represents the shear stress component of T. These stress components can be determined in terms of the constitutive relationships expressing the physical properties of blood.

Particulate phase

The blood cells are assumed to be spherical in shape and are considered to be uniformly distributed. Following assumptions have also been made to make the analysis simple:

(i) Chemical reaction, mass transfer and radiation between the particles and fluid are neglected (ii) The temperature is uniform within the particles (iii) The interaction between particles has not been taken into account.

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Momentum Equations

$$m\left(\frac{\partial w_r}{\partial t} + w_z \frac{\partial w_r}{\partial z} + w_r \frac{\partial w_r}{\partial r} = K(u_r - w_r)\right. \quad (21)$$

$$m\left(\frac{\partial w_z}{\partial t} + w_z \frac{\partial w_z}{\partial z} + w_r \frac{\partial w_z}{\partial r}\right) = K(u_z - w_z) \quad (22)$$

Continuity Equation

$$\frac{\partial N}{\partial t} + \frac{\partial(N w_z)}{\partial z} + \frac{1}{r}\frac{\partial(N r w_r)}{\partial r} = 0 \quad (23)$$

where N is the number density of the particles, w_r and w_z are the components of velocity of particles along r and z directions respectively, m the mass of blood suspended particles and K the Stokes resistance coefficient (which for spherical particles of radius a is $6\pi\mu a$).

Equations (18) and (19) are valid for both Newtonian fluid and non-Newtonian fluid. However, for the flow of blood in large arteries. the mechanical behaviour of blood is very near to that of Newtonian fluid. Thus, if the blood is assumed to be a Newtonian fluid, then equations (18) and (19) reduce to the following equations:

$$\frac{\partial u_r}{\partial t} + u_z \frac{\partial u_r}{\partial z} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u_r}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2}\right] + \frac{KN}{\rho}(w_r - u_r) \quad (24)$$

$$\frac{\partial u_z}{\partial t} + u_z \frac{\partial u_z}{\partial z} + u_r \frac{\partial u_z}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 u_z}{\partial z^2} + \frac{1}{\partial}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right] + \frac{KN}{\rho}(w_z - u_z) \quad (25)$$

where p and ν denote the pressure and the kinematic viscosity of fluid respectively

The Constitutive Equations

Stresses S_ξ, S_θ, p_w and T_w in equations (15) and (16) or equations (18) and (19) are to be determined with the help of the constitutive equations of arterial walls and fluid.

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1 The stress strain relationships of arterial walls.

The arterial tissue is in substance a kind of non-linear viscoelastic material but as per information available in the literature, for the large arteries, the elasticity dominates indeed the non-linear mechanical properties of arterial tissues, whereas the viscosity is a minor effect. As a matter of fact with this in view, if arterial tissue is modelled as a triclinic, transversely isotropic, elastic, incompressible and axisymmetric, material, the non-linear stress strain relatonships of arterial wall can be given by

$$S_i = -p^1 + A_i^j E_j + \bar{B}^{jk} E_j E_k \qquad (i=\xi,\ \theta,\ n) \tag{26}$$

j, k are the indices of sum for midsurface coordinates (ξ,θ,n) (Wu and Lee) S_n is the stress along the normal coordinate to the arc in the meridian direction of the midsurface of the blood vessel, p^1 denotes the hydrostatic pressure of arterial tissue; E_j $(j=\xi,\theta,n)$ denote the physical strains. For a thin wall vessel

$$E_\xi = \frac{\partial v_z}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial v_z}{\partial z} \right)^2 + \left(\frac{\partial v_r}{\partial z} \right)^2 \right] \tag{27}$$

$$E_n = \frac{v_r}{R^*} + \frac{1}{2}\left(\frac{v_r}{R^*} \right)^2 \tag{28}$$

Then the incompressibility condition provides:

$$(1 + 2E_\xi)\ (1 + 2E_\theta)\ (1 + 2E_n) = 1 \tag{29}$$

The coefficients A_i^j , $\bar{B}^{jk}$ $(i, j, k, = \xi, \theta, n)$ are the constants combined by Lame's elastic constants (see [2]). However the hydrostatic pressure p^1 is generally unknown, therefore adding a complemental condition or assumption is necessary for the use of equation (26).

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2. The constitutive equations of fluid.

For an axisymmetric laminar flow, the constituive equations of fluid in cylinderical coordinates can be expressed as

$$T_{zz} = -p + 2\mu \frac{\partial u_z}{\partial z} \quad ; \quad T_{\theta\theta} = -p + 2\mu \frac{u_r}{r} \ ;$$

$$T_{rr} = -p + 2\mu \frac{\partial u_r}{\partial r} \quad ; \quad T_{rz} = \mu \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \qquad (30)$$

where μ is the coefficient of viscosity of fluid. If the blood is assumed to be Newtonian fluid, μ can be taken as a constant or a function of the temperature being independent of the strain rate. However, if necessrary, the blood is also modelled as a Cassonian fluid of non-Newtonian type. In that case, μ is recognised as the apparent viscosity and it can be determined from

$$\mu = \left[(4\, \eta_c\, J_2)^{\frac{1}{4}} + T_c^{\frac{1}{2}} \right]^2 / \, (4\, J_2)^{\frac{1}{2}}$$

$$J_2 = \frac{1}{4}\left[2\left(\frac{\partial u_z}{\partial z} \right)^2 + 2\left(\frac{\partial u_r}{\partial r} \right)^2 + \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right)^2 \right] \qquad (31)$$

η_c denotes the viscosity of Casson, T_c is a quantity equivalent to the yield stress, J_2 is the second order invariant of the strain rate tensor.

The wall stresses of fluid are given by

$$p_w = \left[1 + \left(\frac{\partial R_w}{\partial z} \right)^2 \right]^{-1} \left[-(T_{rr})_w + 2\,(T_{rz})_w \frac{\partial R_w}{\partial z} - (T_{zz})_w \left(\frac{\partial R_w}{\partial z} \right)^2 \right]$$

$$T_w = \left[1 + \left(\frac{\partial R_w}{\partial z} \right)^2 \right]^{-1} \left[-(T_{rz})_w \left\{ 1 - \left(\frac{\partial R_w}{\partial z} \right)^2 \right\} - \left\{ (T_{rr})_w - (T_{zz})_w \right\} \frac{\partial R_w}{\partial z} \right] \qquad (32)$$

where the subscripts w and $(\ )_w$ represent the values of variables at the inner boundary wall.

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Conclusion and Discussion

The equations of motion for the blood vessel and its surrounding tissue as a system are an advantage over earlier results. These equations have been written in terms of the stresses of vessel wall and fluid and the geometry of blood vessel. The stresses of vessel wall are determined by using the constiutive relationships of non-linear elasticity theory developed by Wu and Lee (12). Tne formulation of non-linear viscoelasticity theory could also be used, because the stresses of the vessel wall in the equations of motion are in fact the generalized ones before the substitution by the concrete stresstrain rate relationships of arterial wall. We prefer the former due to complexity in the latter case. Wall stresses of fluid are then formulated by a generalized axisymmetric stress tensor valid to both Newtonian and non-Newtonian fluids. In the equations of motion of the vessel-tissue system, effects of peripheral tissues on the motion of blood vessel are described by the parameters a_i ($i = 1,2, \ldots, 5$). Governing equations of blood flow together with equations of motion of vessel walls are the basic equations of non-linear wave propagation.

The basic equations can be used to solve the problems for the propagation of non-linear pulse waves in arteries together with the consitutive equations of fluid and vessel walls along with relevant initial boundary conditions.

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THE USE OF INTEGRAL TRANSPORMS IN STATISTICS

By

PROFESSOR J. M. C. JOSHI

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Mr. Chairman and Learned Delegates:

It gives me great pleasure to record my intense feeling of appreciation for the nice arrangements we enjoyed with herculean efforts of Professor R. D. Agrawal and his team.

It may be mentioned that Protessor R. D. Agrawal and Professor Gupta who was honoured yesterday for initiating research at S. A. T. I., Vidisha belong to group of workers working on Integral Transforms and Special Functions. A lot of work was done at this centre on these and related topices. The present secretary of Vijnana Parishad of India also belongs to this group.

So it would be in fitness of things if I take this opportunity to elaborate a few Integral transforms and their uses in Statistics. This also fits in the scheme of thing as we are also having a symposium on "Applications of Mathemrtics in Modern Science and Technology" on the occassion.

We would like to say at the outset that one of the purposes of the lecture is to point out that there are many problems which can be beautifully approached by the methodlems of Integral Transforms, and thus there is need for Statisticians to make themselves familiar with this techniqe.

To begin with we define a functional transformation:
Let R and S be two classes of functions. A process which can be performed on any element f of R and yields an element g of S is called a functional transformation and we may write $g = Tf$

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Here class R is the domnin and class S is the range of the transforma-tion.

An important form of the functional transsormation is the one which takes the form

$$F(x) = \int_a^b k(x\,y)\,f(y)\,dy\,.$$

This is known as integral transforn. The function $k(x,y)$ is the nucleus or the kernel of the transform.

Some of the well known integral transforms are:

(i) $k(x,y) = e^{-xy}$ Laplace transform

(ii) $k(x,y) = e^{-ivx}$ Fourier Transform

(iii) $k(x,y) = y^{x-1}$ Mellin Transform

(iv) $k(x,y) = y\,J_n(xy)$ Hankel Transform

(v) $k(x,y) = (x+y)^{-1}$ Stieltjes Transform

(vi) $k(x,y) = \frac{\Gamma(a)}{\Gamma(b)}\,{}_1F_1(a;b;-xy)$ S. M· Joshi tranform

The first three transform have the additional advantage (from the point of view of use in Statistics) that they enjoy the facility of having (a) inversion theorem (b) convolution theorem. These may be stated:

Transform	Inverse transform	Convolution
Laplace	$\frac{1}{2i\pi}\int_{c-i\infty}^{c+i\infty} e^{xv}\,F(x)\,dx$	$f_1 * f_2 = \int_0^\infty f_1(y\,f_2(t-y)\,dy$
Fourier	$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ixv}\,F(x)\,dx$	$f_1 * f_2 = \int_{-\infty}^{\infty} f_1(y) f_2(t-y)\,dy$
Mellin	$\frac{1}{2i\pi}\int_{c-i\infty}^{c+i\infty} y^{-x}F(x)\,dx$	$f_1 * f_2 = \int_0^\infty f_1(y)\,f_2(w/y)\,\frac{dy}{y}$

These classical transfroms have been generalized by replacing the kernel by more general special function or by making the integral in question a multiple integral, by taking the functions $F(x)$, $k(x,y)$ and $f(y)$ vector valued or we have the integral transforms of generalized functions (the so called distributions).

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One of the integral transforms that finds direct application in statistics is the Fourier transform used (by impersonation) as Characteristic functicn.

We know that if $f(y)$ is a p. d. f. that the C. F. is given by:

$$C(t)=E(e^{ity}) = \int e^{ity} f(y)\, dy$$

If C. F. is known we can find p. d. f. via the inversion formula

$$f(y) = \frac{1}{2\pi} \int e^{-ity} C(t)dt$$

Also the convolution operation signifies the fact that the p. d. f. for the sum $W=X+Y$ (where X and Y are two independent random variables with density functions f_1 and f_2) can be represented by it and it is equivalent to the theorem that characterisic function for tne sum of two independent random vaniables is the product of their characteristic functions.

Similarly multivariate Fourier transform and its inverse can be used fruitfully in many problems in Mathematical Statistics.

Unlike the Fourier transform the utility of Mellin transform in statistics is not so well-known. But we observe that Mellin transfrom is a natural tool for tne derivation of the distribution of products and quotients of independent random vaaiables. Epstein [1] and Nair [2] have proved certain results in this direction. The works of Zolotarev [3] Springer and Thompson [4] [5] [6], Fox [7], Subramanian [8], Consul [9] Lominicki [10] and Kotz and Srinivasan [11] may be gone through. To see the power and elegance of Mellin transform as an analytical tool. It can be used successfully to treat the general problem of random variables with negative as well as prositive values [1] The convolution operation in this case is equivalent to the result that the Mellin transform of the product of their Mellin transform we have

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$$M(t)=E(W^{t-1}) = E(\ (XY)^{t-1})=E(X^{t-1})E(Y^{t-1})=m_1\ (t)m_2(t)$$

By using this result we can easily derive the probability density function for the product of two independent Beta-distributed random variables in terms of Gauss Hypergeometric function.

Similerly [7] multivariate Mellin transform and its inversion can be shown to have many applications. For example we may derive the density function for the product $W=XY$ where (X,Y) has a bivariate standardized normal density function with correlation coefficient. The author has recently given [12] an elegant generalization of characteristic function known as S. M. Joshi integral transform (in memory of his reverand Father). Its many possible uses are being currently investigated. This has been obtained by using P. J. operators (named after author's reverand Mother) of fractional integration. Using Fractional calculus an elegant and simple formula, designated as S. M. P. J. formula, for sum of arbitrary powers of first m natural numbers has been developed in [13]. The author has shown use of S. M. Joshi transform in a problem of random walk over a hypersphere [14], [15]. That Statistics can have a profitable application to linguistics was shown in [16] For paucity of time we have not gone into details of calculations.

I again thank the organisers of the Conference and the Symposium for inviting me to address it.

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ABSTRACT OF THE TALK ON ABACUS

By

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Abacus–a beed calculator–is considered one of the best teaching aids for learning numbers and compehending elementary computation. The children love to play with Abacus and the grown ups derives pleasure by sliding the beeds forth and back. Following simple rules of operation of beeds, such as, beeds gain value when pushed up and loose value when slided down, and that each beed in columns from right to left is of value 1, 10, 100, 1000, etc. The numbers are easily represented on Abacus. The place value concept, addition and subtraction of numbers, developing multiple tables, multiplication and division of numbers, square root of numbers, greatest common diviser of numbers, are easily explained to the children, and, in turn, children perfrom the calulations with speed and accuracy. The other advantages of Abacus are that the problems related to currency, weight, measurement, capacity and time are performed directly on Abacus. Some higher concepts like numbers with different bases and mathematical patterns are also explained very satisfactorily with the help of Abacus. Abacus should be used more and more by the school children just like Soroban–a beed calculator is commonly used by the children in Japan.

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ABSTRACTS OF THE PAPERS

PRESENTED IN THE SECOND ANNUAL CONFERENCE OF "VIJNANA PARISHAD OF INDIA" HELD AT S.A. TECHNOLOGICAL INSTITUTE, VIDISHA

(August 18-19, 1990)

PLANE PROBLEMS OF THE THEORY OF ASYMMETRIC ELASTIC CONTINUUM WITH THE MIXED BOUNDARY CONDITIONS

By

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In this paper plane problems of the theory of asymmetric elasticity with mixed boundary conditions have been considered and analysed. To obtain the explicit general solution in this case the tangential traction, normal displacement and rotation potential are prescribed on the circular boundaries of the circular ring. Four particular cases pertaining to the problem of a circular ring supject to (i) tangential traction varying as of cos θ over internal and external boundaries with radial displacement and rotation potential equal to zero, (ii) radial displacement varying as sin θ over the internal and external boundaries with tangential traction and rotation potential equal to zero (iii) uniform internal and external tangential pressures P_1 and P_2 with normal displacement and rotation potenial equal to zero and (iv) uniform internal and external radial displacements wits tangential traction and rotation potential equal to zero are considered. For each case, the stresses, couple-stresses are evaluated numerically. A comparison of stress distribution among asymmetric elasticity, centrosymmetric elasticity and classical elasticity has been done to analyse the resuls.

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ZEITGEBER (POES) MODELLING OF HUMAN RHYTHMS AND SAD

V. N. Sharma, S. Agrawal and A. Saxena
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Light modulates melatonin secretion from the mammalian pineal organ, thereby providing the rest of the body with accurate time signals to influence behavioural and neuroendocrine rhythms. In this paper we discuss our interventions in modelling the suppression of the melatonin release by exposure of bright light, wherefore, it acts as a zeitgeber (synchonizer of circadian cycles). POES is governed by a set of very simple first order differential equations. In this paper the duration, intensity and the wavelength of the light are the parameters included in the modelling of the melatonin suppression emphasizing the transferability characteristics. POES equations may express both almost periodic and apparently chaotic or stochastic behaviours.

A MAXIMUM ENTROPY ANALYSIS FOR $M^x/G/1$ QUEUEING SYSTEM AT EQUILIBRIUM

Madhu Jain
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The principle of maximum entropy (PEM) which is frequently used in information theory can be applied to construct most uncertain probability distribution subject to some constraints expressed by mean values. In this paper, we have suggested paobability distribution of possible states for $M^x/G/1$ Queueing systems in a maximum entropy condition subject to the expected number of customers. The expression for probability distribution involves mean arrival rate, and first two moments of group size and service time distributions. We have established connections with classical queueing theory and operational analysis.

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FEW APPLICATIONS OF MATHEMATICS IN POWER SYSTEM ENGINEERING

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The paper deals with the applications of mathematics in power system Engineering considering generation and transmission systems. Its importance in determining ABCD constants, forming network equations and fault calculations has been emphasised. In brief the applications of computer in short circuit studies, load flow studies and study of transient stability for large and small disterbances have been dealt to enlihten the scope of mathematics in ower system Engineering.

SOLVABLE POTENTIALS FOR SCHLAFLI'S CANONICAL FORM OF RICCATI'S AND MATHIEU DIFFEFRENTIAL EQUATIONS

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Department of Mathematics, Govt. Science College, Rewa

In this paper a few solvable potentials for the Schrödinger equation have been constructed by using Schlafli's canonical form of Riccati's and Mathieu differential equations following the method of Bhattacharjie and Sudarshan (1962). Similar potentials have been shown to be obtained by the method formulated by Bose (1963). Solution and S-matrix of one of the potential derived from Schlafli's canonical form of Riccati's differential equation has also been discussed.

APPLICATIONS OF FRACTIONAL CALCULUS TO CERTAIN ANALYTIC FUNCTIONS

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The author presents some application of Fractionl integral and fractional derivative to some classes of analytic univalent functions.

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ON A FAMILY OF GENERALIZED POLYNOMIAL SYSTEMS ASSOCIATED WITH LAURICELLA FUNCTIONS F_B, F_C AND F_D

By

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Earlier (1990), we have defined a generalized multivariate polynomial system $B_n^A\{(x_m), y\}$ by means of a generating relation involving the generalized hypergeometric function ${}_pF_q$ and the Lauricella function F_A. In this paper we have given the other three generalized polynomial systems involving the Lauricella functions F_B F_C and F_D. After giving their explicit forms and generalized Lauricalla forms we enunciate their n^{th} differential formulae.

THE MULTIVARIABLE H-FUNCTION OF SRIVASTAVA AND PANDA, AND ITS AP LICATIONS IN A PROBLEM INVOLVING LAPLACE EQUATION

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In the present paper, we evaluate interesting integrals involving H-function of Srivastava and Panda and make their applications to derive the solutions of a problem involving Laplace equation. Finally we also derive three expansion formulae involving above multivariable H-function.

* S. A. T. I., Vidisha, M. P., India

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ZEITGEBER (POES) MODELLING OF HUMAN RHYTHMS AND SAD

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ABSTRACT

Light modulates melatonin secretion from the mammalian pineal organ, thereby providing the rest of the body with accurate time signals to influence behavioural and neuroendocrine rhythms. In this paper we discuss our interventions in modelling the suppression of the melatonin release by exposure of bright light, wherefore it acts as a zeitgeber (synchonizer of circadian cycles). POES is governed by a set of very simple first order differential equations. In this paper the duration, intensity and the wavelength of light are the parameters included in the modelling of the melatonin suppression emphasizing the transferabllity characteristics POES equations may express both almost periodic and apparently chaotic or stochastic behaviours.

HAGEN-POISEUILLE FLOW OF A SUSPENSION IN LIQUID

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Shri G. S. Institute of Technology and Science
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In the present paper the Saffman's equations for the flow of dusty gas are modified to consider the flow of liquid suspenision by including the buoyancy fore and increase in the viscosity of liquid due to the presence of dust particle in it. These equations are then used to discuss poiseuille flow of liquid suspension between two stationary parallel plates under uniform pressure gradient. It is found that velocity of liquid suspension is slightly less than the velocity of clean fluid. The velocity of dust praticles is found to depend on the radius and the density of material of dust particles as well as viscosity and density of liquid.

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CUBIC SPLINE APPROACH TO THE STUDY OF RADIAL HEAT FLOW IN ANNULAR SKIN AND UNDERLYING TISSUES OF HUMAN LIMBS

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Jiwaji University, Gwalior-474011

In this phper a radial heat flow problem in skin and underlying tissues of annular regions resembling limbs have been investigated for a one dimensional steady state case. The region has been divided into three annular layers namely epidermis, dermis and subdermal tissues. The mathematical model incorporates the variation in blood flow with respect to postion, metabolic activity which varies with respect to position and tissue temperature; and perspiration. Cubic spline approach has been employed to solve the mathematical model and obtain numerical results. This approach has been compared with other methods of solution

HYDROMAGNETIC STABILITY OF STRATIFIED SHEAR FLOWS

By

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In this paper instability of flow of an inviscid, incompressible, hydromagnetic fluid of variable density between two parallel planes in presence of parallel magnetic field has been discussed. We have reduced the region of unstable modes obtained earlier for the same problem for the case of weak applied magnatic field

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MAGNETO-THERMOSOLUTAL CONVECTION IN AN ELASTICO-VISCOUS FLUID

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Department of Mathematics. Government College, Bilaspur-174001, India.

The thermosolutal convection in an electrically conducting layer of Oldroyd fluid heated from below and subjected to a stable solute gradient is considered in the presence of a uniform horizontal magnetic field. For the case of stationary convection the Oldroyd fluid behaves like a Newtonian viscous fluid. The magnetic field and stable solute gradient are found to have stabilizing effects on the system. The sufficient conditions for the non-existence of overstability are obtained.

DISCOVERY OF ZERO-A SPIRITUAL BACKGROUND

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In this paper we Propose to present the spiritual background to the discovery of Zero and various names used to denote Zero in Vedic literature with the double meaning. We would appreciate India's invention of zero and her contribution to Mathematical Knowledge. We shall also study afresh the Hindu numerical notations and decimal scale; and see how the numerals known as Hindu figures slowly marched to the west through Egypt and nothern Arabia.

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FIXED POINT THEOREM FOR THREE DENSIFYING MAPPINGS IN BOUNDED COMPLETE METRIC SPACE

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In this paper the results of Fisher (1978), Khan (1978), Furi and Vignoli (1969), Iseki (1974) and two recent papers of Pande (1990) have been extended to a more general case in bounded complete metric space. Also in the present paper we have obtained the fixed point theorem for three densifying mappings. Eor related results we refer to Ciric (1976) and Yeh (1978).

A THEOREM ON A SYSTEM OF DIFFERENTIAL EQUATIONS

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Department of Mathematics, B. D. College, Bihiya (Bihar)

This paper presents the constructions of bilinear concomitant and Wronskian associated with a system of simultaneous differential equations and an associated theorem on the bilinear concomitants of the solutions of the system.

ON SOME NEW RESULTS INVOLVING DOUBLE AND TRIPLE HYPERGEOMETRIC FUNCTION

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ABSTRACT

The aim of this paper is to establish some new generating relations for certain hypergeometric functions of two and three varlebias.

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A NOTE ON THE DEGREE OF APPROXIMATION TO CONTINUOUS FUNCTIONS

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A serics Σa_n with the squence of partial sums $\{s_n\}$ is said to be almost Nörlund summable to s, if

$$N_{n,p} = \frac{1}{p_n} \sum_{k=0}^{n} p_{n-k} s_{k,p} \to s,$$

as $n \to \infty$, uniformly with regard to p, where

$$s_{k,p} = \frac{1}{k+1} \sum_{v=p}^{k+p} s_v,$$

and $\{p_n\}$ is a sequence of positive constants such that

$$P_n = \sum_{k=0}^{n} p_k \to \infty \text{ as } n \to \infty.$$

Let Fourier series for 2π-periodic $f \in L[0,2\pi]$ be given dy

$$f(x) \sim a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Assuming $\{p_n\}$ non $\uparrow$ and $\{p_n\}$ non $\downarrow$ for non-increasing and non-decreasing sequences, respectively, we establish several theorems in this note. The first two theorems are the following:

Theorem 1. Let $w^*(t)$ be a non-negative and non-increasing function defined in the right hand neighbourhood of $t=0$. Suppose that $\frac{w^*(t)}{t^{-\mu}}$ is decreasing for some μ, $0<\mu<1$. Let $w(t)$ be the modulus of continuity for a 2π-periodic $f(x)$. Then if $w(t)=o(w^*(t))$, as $t \to 0^+$, we have by almost Nörlund means.

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$$\max_{0 \leqslant x \leqslant 2\pi} \left\| N_{n,p}(f, x) - f \right\| = (w^*(p_n/p_n)).$$

where the sequence $\{p_n\}$ non $\uparrow$ such that condition (2.2) holds.

Theorem 2. Let $w(t)$ be the modulus of continuity of f satisfying the conditions

$$\int_t^\pi \frac{w(t)}{t^2}\, dt = o(H(t)),\ H(t) \geqslant 0.$$

and $\int_0^t H(t)\, dt = o(t H(t))$, as $t \Rightarrow 0^+$

Let $\{p_n\}$ non $\uparrow$ such that (2.2) holds. Then by almost Nörlund means we have $\max_{0 \leqslant x \leqslant 2\pi} \left| N_{n,n}(f, x) - f \right| = o\left(\frac{1}{n} H(\pi/n)\right).$

Condition (2.2) is: If the sequence $\{p_n\}$ non $\uparrow$ then

$$\sum_{k=0}^{n} \frac{p_{n-k}}{k+1} = o\left(\frac{p_n}{n}\right).$$

SOME INTEGRAL REPRESENTIONS OF A GENERALIZED POLYNOMIAL SET $\left\{ M_n^*(x,y) \right\}$

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The present paper deals with integaral representations of the polynomial set $\left\{ M_n^*(x,y) \right\}$. We have obtained finite single, infinite single and finite double integral representations for $\left\{ M_n^*(x,y) \right\}$ in terms of theorems and corollaries. Some special interesting cases have also been derived.

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A NOTE ON INVERSION AND UNIQUENESS THEORMS FOR GENERALIZED MEHLER-FOCK TRANSFORMATION

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Department of applied Mathematics

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Area of work regarding the extension of classical integral transforms to the generalized functions have been playing an important role since the last three decades Such as Zemanian generalized the Laplace and Mellin transformations [1], K-transformation [2], Hankel transformation [3]. Weirstrass transformation [4], Pandey J. N. extended some trnsforms in t to the generalized functions in [5], [6], [7], [8] and [9]. Choudhary M. S. [10] has extended the convolution of Laplace and Hankel transforms in his thesis. Prasad [11] has extended the Kantorouich-Lebdev transformation in to the generalized function.

In the present paper the inversion and uniqueness theorems for generalized Mehler-Fock trnsforn have been investigated. These theorems are essential inorder to genralise classical Mehler-Fock transformation and finding the solution of boundary value problems in theory of elasticity.

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SOME RESULTS FOR MAPS WITH NON-UNIQUE FIXED POINT

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In the present paper, we improve the results of Ciric [2] and Dhage [3] on fixed points in orbitally complete metric spaces and also obtain the localized version of our results.

बहुमानी प्रतिचित्रिणों के पुनरावृत्तिकों का अभिसरण

श्याम लाल सिंह एवं उमेशचन्द्र गैरोला
गणित विभाग गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार २४६४०४

हाल ही में कुर्पिटग में बहुमानी संकारकों के लिये मान पुनरावृत्तिक विधि का अध्ययन किया। यह सुज्ञात है कि यदि सकारक T किसी निश्चित सकुचन शर्त को सन्तुष्ट करे एवं इसिकावा पुनरावृत्तिकों का अनुक्रम अभिसारी हो तो यह T के स्थिर बिन्दु पर अभिसरित होता है। इस प्रपत्र में सिंह द्वारा बहुमानी प्रतिचित्रणों के लिये पारिभाषित इसिकावा पुनरावृत्तिक विधि के अधीन दो बहुमानी प्रतिचित्रणों के लिये स्थिर बिन्दु प्रमेय स्थापित हैं। प्राप्त परिणाम रीअडेस्, नैम्पल्ली–सिंह व अन्य के परिणामों को भी विस्तारित करते है।

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SOME FIXED POINT THEOREMS FOR A PAIR OF MAPPINNGS

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and

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Dass and Gupta [1] studied the fundamental properties of Banach contraction principle in a metric space. Subsequently several mathematicians investigated conditions under which a pair of mappings may have a fixed point see for instance, Chatterji [2], Bhagawat and Singh [3].

Chatterji [2] used this idea to establish some fiixed point theorems for pair of continuous self mappings of a metric space by generalizing the main result of Dass and Gupta [1]. In this paper, we prove some fixed point theorems for pair of mappings, which extend the results of Bhagwat and Singh [1].

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FIXED POINT THEOREM FOR THREE DENSIFYING MAPPINGS IN BOUNDED COMPLETE METRIC SPACE

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In this paper the results of Fisher (1978), Khan (1978) Furi and Vignoli (1969), Iseki (1974) and two recent papers of Pandey (1990) have been extended to a more general case. Also in the present paper we have proved the theorem for three densifying mappings. For related results, we refer to Ciric (1976) and Yeh (1978). The measure of non-compactness 'α' of bounded sets in a metric space was introduced by Kuratowski (1966). For many useful properties about measure of non-compactness 'α' of bounded sets in a metric space, see Nussbaum (1971). The main idea of densifying mapping was introduced by Furi and Vignoli (1969)

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MULTIPLE HYPERGEOMETRIC FUNCTION OF SRIVASTAVA AND DAOUST AND ITS APPLICATIONS IN TWO BUUNDARY VALUE PROBLEMS

By

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In the present paper first we evaluate an interesting integral involving multiple hypergeometric function of Srivastava and Daoust [*Publ Inst. Math.* (Beograd) (N. S.). **9** **(23)** (1969), 199–202, *Nederl. Acad. Wetensch, Proc. Ser. A* **72** = Indag Math. **31** (1969), 449–457; *Math. Nachr.* 53 (1972), 151–159] and then we make its applicattons to solve two boundary value problems on (i) heat conduction in a rod (ii) deflection of a vibrating string, under certain conditions and to establish an expansion formula involving above multiple hypergeometric function.

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A REMARK ON

"HYPERGEOMETRIC FUNCTIONS OF FOUR VARIABLES (I)"

By

Chhaya Sharma and C. L. Parihar

(*Indian Acad. Math.* **11**, No. 2 (1989), 121–133

By

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In this remark, we have called the attention of authors, readers (and the editors) towards the fact that following hypergeometric functions of four variables introduced in the above paper, had already been introduced seventeen years back in the literature by H. Exton ([1], [2], [3]) in different notations :

(1) $F_9^{(4)}(a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1 ; c_1, c_2, c_3, c_1 ; x, y, z, t)$
$= K_1(b_1, b_1, b_1, b_1 ; a_1, a_1, a_1, a_2 ; c_1, c_2, c_3, c_1 ; x, y, z, t),$

(2) $F_1^{(4)}(a_1, a_1, a_1, a_2, b_1, b_1, b_1, b_1 ; c_1, c_2, c_3, c_4 ; x, y, z, t)$
$= K_2(b_1, b_1, b_1, b_1 ; a_1, a_1, a_1, a_2 ; c_1, c_2, c_3, c_4 ; x, y, z, t),$

(3) $F_{38}^{(4)}(a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1 ; c_1, c_2, c_1, c_2 ; x, y, z, t)$
$= K_3(b_1, b_1, b_1, b_1 ; a_1, a_1, a_2, a_2 ; c_1, c_2, c_2, c_1 ; x, y, t, z),$

(4) $F_{10}^{(4)}(a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1 ; c_1, c_2, c_1, c_3 ; x, y, z, t)$
$= K_4(b_1, b_1 b_1, b_1 ; a_1, a_1, a_2, a_2 ; c_1, c_2, c_3, c_1, ; x, y, t, z),$

(5) $F_2^{(4)}(a_1, a_1, a_2, a_2, b_1, b_1, b_1, b_1 ; c_1, c_2, c_3, c_4 ; x, y, z, t)$
$= K_5(b_1, b_1, b_1, b_1 ; a_1, a_1, a_2, a_2 ; c_1, c_2, c_3, c_4 ; x, y, z, t),$

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(6) $F_{59}^{(4)}(a_1, a_1, a_2, a_3, b_1, b_1, b_1, b_1; c_1, c_2, c_1, c_2; x, y, z, t)$
$= K_6(b_1, b_1, b_1, b_1; a_1, a_1, a_2, a_3; c_2, c_1, c_1, c_1; y, x, z, t),$

(7) $F_{39}^{(4)}(a_1, a_1, a_2, a_3, b_1 b_1, b_1, b_1; c_1, c_2, c_1, c_2; x, y, z, t)$
$= K_7(b_1, b_1, b_1, b_1, a_1, a_1, a_2, a_3; c_1, c_2, c_1, c_2; x, y, z, t),$

(8) $F_{11}^{(4)}(a_1, a_1, a_2, a_3, b_1, b_1, b_1, b_1; c_1, c_2, c_1, c_3; x, y, z, t)$
$= K_8(b_1, b_1, b_1, b_1; a_1, a_1, a_2, a_3; c_1, c_2, c_1, c_3; x, y, z, t),$

(9) $F_{12}^{(4)}(a_1, a_1, a_2, a_3, b_1, b_1, b_1 b_1; c_1, c_2, c_3, c_3; x, y, z, t)$
$= K_9(b_1, b_1, b_1, b_1; a_1, a_1, a_2, a_3; c_1, c_2, c_3, c_3; x, y, z, t),$

(10) $F_3^{(4)}(a_1, a_1, a_2, a_3, b_1, b_1, b_1, b_1; c_1, c_2, c_3, c_4; x, y, z, t)$
$= K_{10}(b_1, b_1, b_1, b_1; a_1, a_1, a_2, a_3; c_1, c_2, c_3, c_4; x, y, z, t),$

(11) $F_{60}^{(4)}(a_1, a_2, a_3, a_4, b_1, b_1, b_1, b_1; c_1, c_1, c_1, c_2; x, y, z, t)$
$= K_{11}(b_1, b_1, b_1, b_1; a_1, a_2, a_3, a_4; c_1, c_1, c_1, c_2; x, y, z, t),$

(12) $F_{40}^{(4)}(a_1, a_2, a_3, a_4, b_1, b_1, b_1, b_1; c_1, c_1, c_2, c_2; x, y, z, t)$
$= K_{12}(b_1, b_1, b_1, b_1; a_1, a_2, a_3, a_4; c_1, c_1, c_2, c_2; x, y, z, t),$

(13) $F_{13}^{(4)}(a_1, a_2, a_3, a_4, b_1, b_1, b_1, b_1; c_1, c_1, c_2, c_3; x, y, z, t)$
$= K_{13}(b_1, b_1, b_1, b_1; a_1, a_2, a_3, a_4; c_1, c_1, c_2, c_3; x, y, z, t),$

(14) $F_{77}^{(4)}(a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_1; c_1, c_1, c_1, c_1; x, y, z, t)$
$= K_{14}(a_1, a_1, a_1, a_2; b_1, b_2, b_3, b_1; c_1, c_1, c_1, c_1; x, y, z, t),$

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(15) $F_{78}^{(4)}(a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_4; c_1, c_1, c_1, c_1; x, y, z, t)$
$= K_{15}(a_1, a_1, a_1, a_2; b_1, b_2, b_3, b_4; c_1, c_1, c_1, c_1; x, y, z, t),$

(16) $F_{79}^{(4)}(a_1, a_1, a_2, a_2, b_1, b_2, b_1, b_2; c_1, c_1, c_1, c_1; x, y, z, t)$
$= K_{16}(a_1, b_1, b_2, a_2; c_1; x, y, z, t),$

(17) $F_{82}^{(4)}(a_1, a_1, a_2, a_3, b_1, b_2, b_1, b_3; c_1, c_1, c_1, c_1; x, y, z, t)$
$= K_{19}^{(4)}(a_1, b_1, b_2, a_2, a_3, b_3; c_1; x, y, z, t),$

(18) $F_{81}^{(4)}(a_1, a_1, a_2, a_2, b_1, b_2, b_3, b_4; c_1, c_1, c_1, c_1; x, y, z, t)$
$= K_{20}(a_1, a_1, b_3, b_4; b_1, b_2, a_2, a_2; c_1, c_1, c_1, c_1; x, y, z, t),$

(19) $F_{83}^{(4)}(a_1, a_1, a_2, a_3, b_1, b_2, b_3, b_4; c_1, c_1, c_1, c_1; x, y, z, t)$
$= K_{21}(a_1, a_1, a_2, a_3; b_1, b_2, b_3, b_4; c_1, c_1, c_1, c_1; x, y, z, t).$

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UNIFIED STUDY OF TWO GENERAL CLASSES OF FUNCTIONS

By

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To unify the study of two general classes of polynomials Chandel and Bhargava [1] introduced the polynomials $G_n(a, k; hg(x))$ defined by

(1.1) $$G_n(a,k;h,g(x)) = e^{-hg(x)} \; T^n_{a,k}\left\{ e^{hg(x)} \right\},$$

where h is consant and $g(x)$ is differentiable function of x. Recently Chandel and Agrawal [2] introduced a class of polynomials defined by Rodrigues' formula

(1.2) $$S_n^{(\alpha,k)}(h,g) = (1-hg)^{\alpha}\, \Omega_x^{m} \{1-hg\}^{-\alpha},$$

where α, h, k are independent of x and g is any function of x, and $\Omega_x \equiv x^k D$.

The motivation of this paper is to unify the study of these two general classes by introducing the sequence of functions

$\left\{ G_n^{(a,b,k)}(h,g(x)) \right\}$ defined by Rodrigues' formula

(1.3) $$G_n^{(a,b,k)}(h,g(x)) = (1-h\,g(x))^b \; T^n_{a,k} \{ (1-hg(x))^{-b} \},$$

where a,b,h,k are independent of x and $g(x)$ is a suitable differentiable function of x and

(1.4) $$T_a \equiv x^k (a+xD).$$

Replacing h by h/b and taking $b \to \infty$, (1.3) reduces to (1·1), while particularly for $a=0$ on replacing k by k-1 and b by α, (1.3) reduces to (1.2).

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FRACTIONAL DERIVATIVES OF CONFLUENT HYPERGEOMETRIC FORMS OF KARLSSON'S MULTIPLE HYPERGEOMETRIC FUNCTION ${}^{(k)}F_{CD}^{(n)}$

By

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Motivated by the earlier work of Srivastava and Goyal [10], recently Srivastava, Chandel and Vishwakarma [11] derived fractional derivatives involving generalized multiple hypergeometric function of Srivastava and Daoust [9] and discussed their special cases to derive the results involving the multiple hypergeometric functions of several variables (and their confluent forms), defined by Lauricella [7] Exton [4,5], Chandel [1], Chandel-Gupta [2] and Karlsson [6].

In the present paper, for special interest we apply same techniques in order to derive fractional derivatives involving confluent hypergeo metric forms ${}^{(k)}_{(1)}\phi_{CD}^{(n)}$, ${}^{(k)}_{(2)}\phi_{CD}^{(n)}$, ${}^{(k)}_{(3)}\phi_{CD}^{(n)}$ and ${}^{(k)}_{(4)}\phi_{CD}^{(n)}$ of Karlsson's multiple hypergeometric function ${}^{(k)}F_{CD}^{(n)}$ [6], recently introduced by the authors [3].

ACKNOWLEDGEMENTS

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VALEDICTORY FUNCTION

(August 19, 1990)

Chief Guest: Professor H. N. Silakari, Principal, S. A. T. I., Vidisha
Chairperson: Professor P. M. Gupta, Retd. Professor of Mathematics S. A. T. I., Vidisha

A REVIEW OF TECHNICAL SESSIONS

Dr. G. C. Sharma

Professor and Head, Department of Mathematics, Agra Universiy, Agra

I have attended four conferences this year out of which two are very important. First one is Ramanujam Society's meeting at Mysore where the Principal, Prof. Maheshwari made some comments and here Principal Prof. Silakari mede the comments. Both these comments were the same: What have we done after Independence in the field of Science and Technology? So this is a question to all scientists present here.

Coming to the First Technical Session, Dr. Narendra Puri explained how "Vedic Mathematics" can play a role in teaching the Number System to young students. Mathematicians consider these to be 'Tricks'. I don't say that tricks are bad, they must be having some system, but the system must be generalised and as commented by Dr. P. L. Sharma there must be some system in this and it should be evaluated. Thereafter Prof. P. L. Sharma gave a very lively talk and for a while the whole of the house was in laughter. He illustrated "Vedantic Principles" by very simple examples to show how sorrow can be reduced with the help of mathematical principles.

In the second Technical Session under the chairmanship of Professor V. P. Saxena, the distinguished scientist Dr. M. K. Jain presented his talk on "Differential Equations". In his presentation he made three points very clear, consistency, stability and convergence of the solutions of Differential Equations. He also covered the most modern computations and the, "Navier Stokes-Solutions for Hyperbolic and Elliptic differential Equations. Dr. D. S. Karaulia gave a talk on "Mathematical

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Logic". It was relevant to "Artificial Intelligence" which is a leading subject now-a-days, in the field of Computer Sciences. This session in fact could be termed as a session of Numerical Engineering.

The next session was on "Mathematical Modelling". In this Dr. V. P. Saxena emphasized the concept of **Precision in Prescribing medicines.** Usually medicines prescribed by a doctor are on empircal-basis. In my own problem I have tried to explain how blood fiow takes place and deduced the equations of the blood-walls, tissues have also been taken into consideration. The classical method has been replaced haer by a system of equations. Under the chairmanship of Dr. Subramanian one more delightful talk was given by Dr. S. N. Pandgy on the "Properties of Matter" as described by the human mind and said that they were very simple to understand the whole Universe. He clarified the concepts of "Hole" and "Gravitational waves".

On the second morning of the conference, Prof. J. M C. Joshi presented a talk on "Integral Transforms", Prof. S. N. Pandey chaired the session. This talk was of great importance so far as the applications to statistics and other areas are concerned. As the Mathematics Department of S. A. T. I., Vidisha is very much popular for Research on "Special Functions" and "Integral Transform", they must have enjoyed it. Moreover in any branch, whosoever is trying the solutions of differential equations 'Integral Transforms' is a technique by which the engineers and mathematicians can find out a close-form solution. A talk on "Fixed Point. Theorem" was given by Dr. S L. Singh. Dr. P. L. Snarma chaired the session. The speaker gave some theorems which enlightened the gathering. Further under the chairmanship of Dr. S. L. Singh one of my colleagues Dr. Sunder Lal gave a talk, and every one of you who were present during his talk could visulize a hidden teacher and a good researcher in him. He explained every thougt of Toplogy in such a simple and lucid manner that even students could understand.

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In the last session, three talks were given. Dr. Narendra Puri talked on "Vedic Mathematics for Engineers". In second talk Dr. Subramanian delighted us by his pictorial presentations. Particularly "Hermite-polynomials" which is the point of interest for those who work in the field of 'Special Functions' and "Vedantic-Relations" in which he related with words every good quality of a person, quoting "Hermite-Polynomials". For example Politeness for P Original for O and so on. We can confidently say that he is a well read person. He has done good research works as well and he can give some message to the society. The third talk of this last session was on "ABACUS" by Dr. R. P. Singh. Listening to this talk one can definitely say that Dr. R. P. Singh has devoted himself sincrely to the cause of Primary Education. As it is the slogan of the country that we should give more stress on Primary Education. A person like Dr. R. P. Singh can attract the teachers to the important task of teaching mathematics. Definitely then Arithmatic from the very begining shall be a point of attraction to the children and when they grow up they will do very good work. That will also be the answer to the questions put forth by Prof. Silakariji in his welcome address.

Undoubtedly, Conference is National.

At last I thank you again to invite me to put my comments before you and if somebody has felt anything otherwise, I shall beg his pardon.

Thank You.

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EXPERT'S COMMENTS ON "PAPER READING SESSION"

Dr. Sunderlal

Reader, Deptartment of Mathematics, Agra University, Agra

Chief Guest Prof. Silakariji, Chairperson Dr. Gupta, Dear friends:

In the paper reading session about 30 papers were received for presentation and out of these 21 were actually read. The time available at our disposal was about one and half hours, though we took more than two hours. As Prof. Singh has already said, during these paper reading sessions we have covered a wide range of topics staring from Ancient Indian Mathematics, Pure Mathematics, Applied Mathematics, Applicable Mathematics and Computer Sience. We had people from different back-grounds, of-course we had mathematicians but we also had people from physics, civil engineering, electrical engineering etc. Definitely the time was short because normally six minutes were available while ten minutes is a reasonable period for a paper presentation. But all the spekers were co-operative and the programme went on very much according to schedule. On their behalf I would like to thank the organisers for giving them this opportunity. I hope in future the *Vijnana Parishad of India* would be able to devote more time for this type of activity. viz. the paper-reading sessions, and we hope that in future we would be attracting more young people for paper presentation.

Finally, I remark that Conference is National because it has participation from all parts of the Country.

Thank you.

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कार्यवाहक अध्यक्ष, भारतीय विज्ञान परिषद

डॉ. एस. एल. सिंह का उद्बोधन

आदरणीय मुख्य अतिथि प्रो० सिलाकारी जी, अध्यक्ष महोदय प्रो० गुप्ता जी, विद्वान साथियों देवियों एव सज्जनों:

अभी-अभी मैंने आज के कार्यक्रम की सूची पढ़ी है। इसके अनुसार संयोजक सचिव डॉ० आर० डी० अग्रवाल ने प्रत्येक व्यक्ति को कम से कम एक बार बोलने का मौका अवश्य दिया है। मेरे विचार में श्रेयस्कर होता कि "पेपर रीडिंग सेशन" के अध्यक्षों द्वारा ही विशेषज्ञ टिप्पणी दी जाती। फिर भी जब मुझे आदेश हुआ है तो मैं कुछ टिप्पणी अवश्य करूंगा, साथ ही अपनी बात आपके सामने रखूंगा।

हालांकि एस. ए. टी. आई. विदिशा में हम अतिथि के रूप में उपस्थित हैं, किन्तु आज हमारे समारोह में इसी संस्था के प्राचार्य प्रो० सिलाकारी जी, मुख्य अतिथि के रूप में मौजूद हैं। मैं "भारतीय विज्ञान परिषद" की ओर से उनका स्वागत करता हूं-अभिनंदन करता हूं।

किसी भी समिति-परिषद के वार्षिक अधिवेशन की सफलता-असफलता मुख्यतया तीन बातों से आंकी जा सकती है। मेरे विचार से वे इस प्रकार हैं। आयोजक संस्था द्वारा स्वागत-सत्कार व्यवस्था, अधिवेशन की कार्यवाही के दौरान समिति के पदाधिकारियों की भूमिका एवं शैक्षणिक सत्र। इन सबमें शैक्षणिक सत्र सबसे महत्वपूर्ण मुद्दा हैं।

"पेपर रीडिंग सेशन" एवं चर्चा में विभिन्न प्रतिनिधियों द्वारा विविध विषयों का समायोजन था। उदाहरण के लिये हमारे विद्वानों ने "लॉजिक" "आर्टी फीशियल इन्टेलीजेन्स" "रिलेटिविटी", "इलास्टीसिटी", "इन्जीनियरिंग", "टोपोलॉजी", "ऑपरेशन थ्योरी" "थ्योरिटिकल फिजिक्स", बॉयो मेडिकल साइन्सेज", "मैथमेटिकल बॉयलाजी", "ऑपरेशन्स रिसर्च" "मैथमेटिक्स एजुकेशन" आदि न केवल पुरातन मौलिक विषयों पर चर्चा की, वरन् वर्तमान में रुचिकर आधुनिक विषयों पर भी विचार-विमर्श किया। इस प्रकार हम कह सकते हैं, कि इस दो दिवसीय सम्मेलन में हमने अनेकानेक विषयों के विस्तृत क्षेत्र को छुआ है भले ही हम विषय की अधिक गहराई में न गये हों।

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प्रो॰ जी॰ सी॰ शर्मा, आर्थिक परिप्रेक्ष्य में "संयुक्त सत्र" के पक्षधर हैं। उनके अनुसार ये कम खर्चीले तो होते ही हैं, साथ ही साथ इनमें अन्तर्विभागीय परस्पर विचार विमर्श का भी अवसर प्राप्त होता है। प्राय: हम अन्तर्विभागीय क्रियाकलापों की वकालत तो करते हैं, किन्तु अधिकतर सम्मेलनों में प्रतिनिधियों को अलग-अलग कई समूहों में (उनके विशेषज्ञता के विषय के अनुसार) बांट कर उनसे अन्तर्विभागीय विचार-विनिमय की अपेक्षा करते हैं। जो कि सर्वथा असंभव है। बड़ी विडम्बना है कि जब-जब भी इस प्रकार अन्तर्विभागीय-विचार-विनमय का मौका आता है, प्रतिनिधियों को विभाजित कर दिया जाता है। अत: मेरे विचार से सम्मेलनों में हमेशा 'संयुक्त- सत्र' ही होना चाहिये, चाहे प्रतिनिधि- संख्या कितनी भी अधिक क्यों न हो।

किसी सम्मेलन के संपूर्ण क्रियाकलापों का विश्लेषण किन्हीं अन्तर्राष्ट्रीय-स्तर के विश्लेषकों से किया जाना उचित रहता है। सौभाग्यवश अन्तर्राष्ट्रीअ-स्तर के एक विश्लेषक प्रो॰ जी॰ सी॰ शर्मा ने अभी-अभी अपने विचार आपके सामने रखे हैं। चूंकि प्रो॰ शर्मा भी भारतीय विज्ञान परिषद के एक अग हैं, अतः संभव है परिषद से लगाव वश उन्होंने इस सम्मेलन की कुछ अतिरिक्त प्रशंसा कर दी हो। अत: मैं निवेदन करना चाहूंगा कि इस सभागार में से कोई भी व्यक्ति आकर २ मिनट में इस अधिवेशन के बारे में अपनी राय व्यक्त करे।

जहाँ तक आयोजक संस्था द्वारा प्रदत्त स्वागत-सत्कार आदि की व्यवस्था का प्रश्न है, उसकी उदारता-सज्जनता एवं सौजन्यता का शब्दों में वर्णन कदापि सभव नहीं है। किसी उपहार हेतु किसी व्यक्ति को धन्यवाद तो दिया जा सकता है उसका आभार माना जा सकता है किंतु आतिथ्य सत्कार में जो गर्मजोशी व अपनापन मिला वह सिर्फ अनुभूति की बात है। जो भावनात्मक लगाव हमने अनुभव किया है उसको शब्दों में लौटा पाना असंभव है। फिर भी औपचारिकता वश कहना आवश्यक है। मैं भारतीय विज्ञान परिषद की ओर से प्राचार्य प्रो॰ सिलाकारी जी, प्रो॰ आर॰ डी॰ अग्रवाल एवं उनके समस्त प्राध्यापक साथियों-कर्मचारियों एवं छात्र-छात्राओं का आभार मानता हूं और धन्यवाद देता हूं।

इस सम्मेलन की सफलता के पीछे आप सभी का व्यक्तिगत लगाव व परिश्रम स्पष्ट दृष्टिगोचर है। मैं विश्वास दिलाता हूं कि भविष्य में जब कभी भी हमें फिर यहाँ बुलाया जायेगा हम सहर्ष यहां आना चाहेंगे और प्रतिनिधियों की संख्या इस

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कहीं अधिक होगी । साथ ही साथ मैं अपने प्रतिनिधि बंधुओं से भी निवेदन करना चाहूंगा कि वे भी यहां से जाकर अपने अनुभव अपने अपने संस्थानों में सुनायें ।

हमारा हमेशा यही प्रयास रहा है कि उपलब्ध संसाधनों का समुचित उपयोग किया जाये । यहां पर हमने थोड़ा सा परंपरा से हटकर कार्य किया है । प्राय: एक सत्र की तीन चार चर्चाओं हेतु केवल एक ही अध्यक्ष रहता है यहाँ पर हमने प्रत्येक चर्चा के लिए एक पृथक अध्यक्ष नियुक्त किया था, क्योंकि गणित के विभिन्न विषयों पर चर्चायें थीं, और उस क्षेत्र के विशेषज्ञ विद्वान को उस वार्ता के लिये अध्यक्ष बनाने से वक्ता के साथ पूरा-पूरा न्याय हुआ है । मेरे विचार से हमारा प्रयास सफल रहा ।

हमें इस सम्मेलन में परिषद के अध्यक्ष डॉ० जे० एन० कपूर साहब की कमी निरन्तर खली है । यदि वे आज यहाँ होते तो इस आयोजन की सफलता पर अति प्रसन्न होते । हम उनके भी आभारी हैं, जो समय-समय पर हमें उनसे मार्गदर्शन और प्रेरणा मिलती रही है ।

अन्त में मैं "भारतीय विज्ञान परिषद" की ओर से सी० एस० आई० आर०, नई दिल्ली, शिव शक्ति पेपर्स मिल्स विदिशा, ब्रिटानिया इंडस्ट्रीज लि० विदिशा, बरकतउल्ला विश्वविद्यालय भोपाल एवं मध्य प्रदेश विज्ञान एवं प्रौद्योगिकी परिषद, भोपाल का भी आभार मानता हूं जिनका इस सम्मेलन की सफलता में विशेष सहयोग रहा है ।

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VALEDICTORY ADDRESS

Professor H. N. Silakari, Principal
S. A. T. I., VIdisha, M. P.

Chairman, Members of the ' Vijnana Parishad of India", Distinguished Members of the panel of Experts, all the Eminent Scholars, my Friends, Collegues and all those present here:

I am really very much thankful for the very nice and generous sentiments expressed by Dr. Singh, Dr. Joshi, Dr. Sharma and others regarding the organisation of the conference and the reception and hospitality extended to the delegates. But I would like to remind them what our Honourable Minister said yesterday about ' SUDAMA'S TANDUL" and "SHABRI'S BER". So we have already apologised, even if the Hon'ble Minister did so on our behalf, to accept whatever little hospitality we were able to offer.

I have been reminded by some of the speakers about certain questions which have been asked by me. The same questions must have been asked by other people at other times. These are some fundamental questions and necessary to be asked at this time among a gathering of scholars, particularly mathematicians.

Being a person who is not directly involved with mathematics, it was only the most fundamental matters about which I would speak and not the intricacies of the mathematical sciences. Those questions have been already before us and I believe that the "Vijnana Parishad of India" is not un-aware of them. As Dr. Singh said these questions will be discussed and definitely our scientific community has to findout an answer to them.

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Yesterday when Dr. J. N. Kapoor's lecture was recited here by Dr. Saxena, one thing which struck me most was the observation that the level of scientific activity or the intensity of scientific activity in our country is far behind the level or intensity of scientific activity in some of these advanced countries. That is something which really must have struck most of us, but I am forced to repeat it because nobody has made any observation avout it. "Vijnana Parishad of India" is an Association which can certainly work in this direction. As Dr. Kapoor has said in his lecture, which was read here, we must increase our activities about 50 timesof their present level. This is something very remarkable.

Another thing which I would like to say at this time before this august gathering of Mathematicians is to do something about mathematical education. How this education of mathematics can be improved so that the 'Phobia' which exists about mathematics is removed particularly in a paofessional college where people have to read a lot of mathematics and it is supposed to be the biggest killer. Certainly a Science should not become a killer. What happens is that people are not excited about Mathematics, they do not adopt it as they should because this is a permanent asset to them even in their professional life. They read it, give it up and go on to other subjects, which appear to them more relevant. But as we know, in all the sciences, not only in mathematics but in all the sciences and technologies the moment you go beyond the most primitive levels you can not do without mathematics, and if a person is unable to adopt it as a way of life, as something which is always there with him, like a tool just as a farmer will have his sickle with him he can not go very far in his field of work. He must have that tool with him and he must feel one with it. That is a problem about which the mathematicians have to think-over. It is not only that this is a problem in mathematics, the problem of education is

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there in all the fields, but I am refering to it here because mathematics happens to be a very important field and because in an Engineering College, we are coming across this type of problem too frequently and unless the engineers can adopt this tool of mathematics whole-heartedly, can make use of it with ease and have confidence in it, then perhaps, we are failing in our obiectives.

I do not think that I should take more of your time and talk about other matters. I thank you all for coming-over here at our invitation.

As Dr. Singh has said, whenever we invite you again, you will be very happy to come-here. In fact that may be one of the reasons that we have extended some hospitality to you. Because we would like to have our scholars and mathematicians and the members of Vijnana Paaishad who hnve come here, the experts who have come here at our invitatian, to come here more often and we would like to provide occassions for it in future so that we have the pleasure of your company again and again.

Thank You.

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समापन समारोह

अध्यक्षीय भाषण

डॉ० पी० एम० गुप्ता, पूर्व विभागाध्यक्ष, गणित विभाग
एस० ए० टी० आई०, विदिशा

मुख्य अतिथि प्राचार्य सिलाकारी जी, डॉ० अग्रवाल, विशेशज्ञ समिति के सदस्यगण, परिषद के पदाधिकारीगण उपस्थित सज्जनों एवं देवियों।

मैं इस महाविद्यालय में सन् १९६२ से १९८२ तक रहा था। जिस रिसर्च रूपी पौधे को मैंने यहाँ रोपा था उसे फलता-फूलता देखकर मुझे अत्यन्त प्रसन्नता हो रही है।

सर्वप्रथम सन् १९७२ में इस सस्था के गणित विभाग के तत्वावधान में "Indian Mathematical Society" का एक दिवसीय अधिवेशन हुआ था। इस अधिवेशन में "भारतीय विज्ञान परिषद" के वर्तमान अध्यक्ष डॉ० जे० एन० कपूर, उपाध्यक्ष डॉ० एम० के० सिहल एवं सचिव डॉ० आर० सी० सिंह चन्देल भी शामिल हुये थे। आज के कार्यक्रम के मुख्य अतिथि और इस संस्था के प्राचार्य प्रो० सिलाकारी जी, तब हमारे सहकर्मी थे।

किसी भी आयोजन की सफलता में आयोजक संस्थान के प्रमुख की अहम् भूमिका रहती है एवं उनका सहयोग परम आवश्यक होता है। आज यहां पर भी "भारतीय विज्ञान परिषद" के द्वितीय वार्षिक अधिवेशन एव सम्मेलन की सफलता का श्रेय प्रो० सिलाकारी जी, को देता हूं। इस महान सफलता हेतु मैं डॉ० आर० डी० अग्रवाल, गणित विभाग के समस्त प्राध्यापकगण एव इस संस्थान के उन सभी कर्मचारियों, शिक्षकों को धन्यवाद ज्ञापित करता हूं, जिनका इस कार्यक्रम की सफलता प्रदान करने में प्रत्यक्ष या परोक्ष सहयोग रहा।

इस संस्थान का शुभारम्भ सन् १९६१ में हुआ था और उस समय यह महाविद्यालय विक्रम विश्व विद्यालय उज्जैन से सबद्ध था। विदिशा की तुलना में अन्य बड़े व पुराने शहरों की संस्थायें इन्दौर, भोपाल, ग्वालियर, उज्जैन आदि भी इसी विश्वविद्यालय से संबद्ध थीं। फिर भी हमने विश्वविद्यालय प्रशासन में अपनी प्रभुता बनाये रखी थी, और इसी प्रभुता व परस्पर सहयोग के बल पर वरिष्ठ साथियों को सहायता से 'एम० एस० सी० की संबद्धता प्राप्त की और उसे आज तक जारी रखा है।

हमने अपना 'रिचर्स कार्यक्रम' ,इन्टीग्रल ट्रान्सफार्म' स प्रारंभ किया था और बाद में 'ऑपरेशन्स रिचर्स' के क्षेत्र में भी कार्य किया। डॉ० आर० डी० अग्रवाल आज भी उस रिचर्स कार्यक्रम को सतत जारी रखे हुए हैं। इस सस्थान को निरंतर प्रगति पथ पर अग्रसर देखते हुए मैं आनंदित हूं।

मेरी शुभकामनायें हैं कि यह संस्थान तथा "भारतीय विज्ञान परिषद" उन्नति के चरमोत्कर्ष पर पहुंचे।

धन्यवाद !

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CONCLUDING SPEECH BY PARISHAD SECRETARY

Dr. R. C. Singh Chandel
Secretary, Vijnana Parishad of India
D. V. Postgraduate College, Orai–285001, U. P.

Hon'ble Chief Guest Principal Silakariji, Chairman Prof. P. M. Guptaji, Organising Secretary Dr. R. D. Agrawal, Delegates and my Brothers and Sisters:

I am very much thankful to all of you who accepted our invitation, gave your valuable time and took active part in the conference and NATIONAL SYMPOSIUM on "APPLICATIONS OF MATHEMATICS IN MODERN SCIENCE AND TECHNOLOGY", especially to Prof. M. K. Jain, Prof. P. M. Gupta, Prof. G. C. Sharma, Prof. S. N. Pandey, Prof. P. L. Sharma, Prof. R. P. Singh, Prof. J. M. C. Joshi, Prof. V. P. Saxena, Prof. Narendra Puri, Prof. Subramanian, Prof. Sunderlal, Prof. Ghosh, Prof. S. L. Singh and Prof. D. S. Karaulia. I am thankful to Protessor J. N. Kapur (President), who is at present visiting Canada but he continuausly guided us from there.

I am also thankful to Honourable Shri Mahesh Prasad Shukla, Minister of State, Department of Manpower Planning, Govt. of Madhya Pradesh, who aecepted our invitation and spared his valuable time to inaugrate the Second Annual Conference of VIJNANA PARISHAD OF INDIA. We are also thankful to Shri Raghavji Bhai, M. P., Chairman, Administrative Committee, S. A. T. I., Vidisha for chairing the Inaugral Function and to Shri Chhabi Ram Argal, M P., Morana and to Shri Rajendra Sharma, M. P., who came from Delhi as special guests to take active part in the Conference and Symposium (Specially in Vedic Mathematics and its Applications). I also wish to pay my regards with sincere feelings of gratitude to my guide Prof. P. M. Gupta, (Chair person), who came here from Delhi as a token of

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his kind love for us. I am specially thankful to Hon'ble Principal Silakariji and Local Organising Secretary Dr. R. D. Agrawal who accepted our proposal for holding the conference here It is only due to their keen interest that the conference could be held successfully. The arrangement of lodging, boarding, and convence was quite nice and for which I am very much thankful to the whole team of workers of S. A.T. I., Vidisha.

I am thankful to C. S. I. R., New Delhi, S. A. T. I. Vidisha, Shivshakti Paper Mills, Vidisha, Britannia Industries Ltd. Vidisha, Barkatullah Vishwavidyalaya, Bhopal and the M. P. Council of Seience and Technology, Bhopal, who sponsored the confernce and shared its financial burden.

I hope all participants, guests, local organisers will excuse me for any mistake committed from the side of "Vijnana Parishad of India".

Form all parts of the Counpry 74 participants attended the Conference and during very busy programme of two days, 13 invited talks were given by distinguished Scientists and 21 papers were presented. I hope that the recommendations of the Conference shall be implemented by your Institutions, and Universities. We assure you that Parishad will make its full efforts to see that the recommendations of the conference are implemented by U. G. C., C. S. I. R., D. R. D. O., and other agencies of the Government of India.

We hope our participants will cary the Torch of the Parishad atleast to their own areas and regions and make mathematics truly applicable at all levels from primary education to High Class Research in all Engineering as well as Social, Physical, Biological, Medical and Computer Sciences.

We are sure there will come a day when our Parishad will show a torch to light the whole world.

I thank you once again all brothers and sisters who came here from all parts of the country, and took active part in the conference to make it successful

JAI GANITA, JAI PARISHAD and JAI HIND.

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CONCLUDING SPEECH BY LOCAL SECRETARY

Dr. R. D. Agrawal, Professor and Head
Departement of Mathematics and Computer Applications
S. A. T. I., Vidisha, M. P.

A concluding session is very painful. Yesterday and day-before yesterday when I was receiving you all, I was over-whelmed with joy and enthusiasm. Departing today is very painful, especially with intellectuals of the same field.

I am very much thankful to you all for coming over here bearing all the discomforts of the journey and heavy rains. It is the courtesey of you all, that you have not complained about any arragement. It is for you all to comment on the success or otherwise of the conference.

We had at S. A. T. I., a one day session of "Indian Mathematical Society" In 1972 as mantioned by Dr. P. M. Gupta. After a lapse of 18 years we have been able to organise an activity in Mathematics Department.

I wish to accord thanks to all the invited speakers, who have accepted our request at such a short notice and co-operated with us in all respects. I also thank all the participants who have read their papers and wish them a successful future.

I wish to record my sincere thanks to M/s Britannia Industries Ltd., and M/s Shivshakti Paper Mills, Vidisha for providing lunch to participants and guests on August 18 and 19 respectively. On behalf of the organising committee I wish to thank the Mathematics Department of S. S. L. Jain College, Vidisha for providing 'Tea' on 18th August.

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I thank the M. P. Council of Science and Tehnology, Bhopal and Barkatullah University, Bhopal for providing financial assistance.

I shall fail in my duties if I do not record my appreciation for our dynamic and worthy Principal for encouraging me and for providing all facilities and co-operation, without this it would not have been possible to cope-up with the gigantic task.

Last but not the least I am thankful to Prof D. S. Joshi, Dr, D. P. Shukla, Dr. D. P. Goyal, Dr. R. C. Jain, Prof. Dubey, Prof. Swami, Prof. J. K. Jain. Prof. M. K. Choudhary, Prof. R. C. Jain, Prof. S C. Mathur, Prof. Suresh Jain, Prof. S. K. Gour, Prof. K. K. Punjabi, Prof. Lokesh Bajpai, and Prof. Sandeep Jain for their kind co-operation. Shri C. G. Pillai, Shri Jagdish Gupta, Shri Sikarwar, deserve thanks for their sincere help in the various important tasks without which a conference on such a large scale can not be organised.

I wish you all a happy journey back.

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REPORT

THE SECOND ANNUAL CONFERENCE OF
VIJNANA PARISHAD OF INDIA

AUGUST 18, 1990

INAUGURAL FUNCTION: (10.00 A. M.)

Chief Guest: Hon'ble Shri Mahesh Prasad Shukla Minister of State,
Department of Man Power planning
Government of Madhya Pradesh

Chairperson: Shri Raghavji Bhai, M. P.,
Chairman, Administrative Committee
S. A. T. I., Vidisha

Special Guests: Shri Chhabi Ram Argal, M. P., Morana
Shri Rajendra Sharma, M. P.

TEA-BREAK

SYMPOSIUM: AP LICATIONS OF MATHEMATICS IN MODERN SCIENCE AND TECHNOLOGY

First Technical Session: (11.45 A. M.)

Chairperson: Dr. M K Jain, Vice-Chancellor
A P. S. University, Rewa

Talks

1. "Vedic Mathematics and its Applications"
by Dr. Narendra Puri, University of Roorkee, Roorkee
2. "Ganita Ke Beej-Mantra" (Hindi)
by Dr. P. L. Sharma, Prof. and Head
Sagar University, Sagar.

LUNCH (1.00 P. M.)

Second Technical Session: (2.15 P. M.)

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Chairperson: Dr. V. P. Saxena, Prof. and Head
Jiwaji University, Gwalior

Talks

1. "Numerical Solutions of Partial Differential Equations"
By Dr. M. K. Jain, Vice-Chancellor
A. P. S. University, Rewa
2. Underlying Mathematical Elements of Some AI Applications by Dr. D. S. Karaulia, T. T. T. I., Bhopal

TEA-BREAK

Third Technical Session: (3.30 P. M.)

Chairperson: Dr. G. C. Sharma, Prof. and Head
Agra University, Agra

Talk 1. "Bio-Mechanics in Cancer"
by Dr. V. P. Saxena, Prof. and Head
Jiwaji University, Gwalior

Chairperson: Dr. R. P Singh, Field Adviser, N. C. E. R. T.
Regional College of Education, Bhopal

Talk 2. "Non-Linear Wave Propagation in Arteries"
by Dr. G. C. Sharma, Agra University, Agra

Chairperson: Dr. P. R. Subramanian, Prof. of Nuclear Physics
Madras University, Madras

Talk 3 "Some Considerations Based on Gravitational waves"
by Dr. S. N. Pandey, Prof. and Head
M. M. M. Engg. College, Gorakhpur

DINNER (8.00 P. M.)

AUGUST 19, 1990

SYMPOSIUM (Contd.)

Fourth Technical Session: (8.00 A. M.)

Chairperson: Dr. S. N. Pandey Prof. and Head
M. M. M. Engineering College Gorakhpur

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Talk 1. "Use of Integral Transforms in Statistics"
by Dr. J. M. C. Joshi, Prof. and Head
Kumaun University, Nainital

Chairperson: Dr. P. L. Sharma, Prof. and Head
Sagar University, Sagar

Talk 2. "Coincidences and Fixed Points"
by Dr. S. L. Singh, Prof. and Head
Gurukul Kangri University, Haridwar

Chairperson: Dr. S. L. Singh, Prof. and Head
Gurukul Kangri University, Haridwar

Talk 3. "Some Normality Conditions in Topological Spaces"
By Dr. Sunder Lal, Reader, Agra University, Agra

TEA-BREAK

Fifth Technical Session: (10.00 A. M.)

PAPEPS READING SESSION

Chairperson: Dr. Sunder Lal, Reader, Agra University, Agra

1. A remark on "Hypergeometric functions of four variables (I)"
By Chhaya Sharma and C. L. Parihar
(C J. Indian Acad. Math , 11 No 2 (1989), 121-133)
R. C. Singh Chandel and Hemant Kumar,
D. V. Postgraduate College, Orai 285001, U. P.
2. The Multivariable H-function of Srivastava and Panda and its applications in a problem involving Laplace equation,
R. C. Singh Chandel, R. D. Agrawal(2) and Hemant Kumar
D. V. Postgraduate College, Orai (2) S. A. T. I., Vidisha, M. P.
3. Plane problems on the theory of asymmetric elastic Centinuum with the mixed boundary conditions, S. K. Ghosh, Flexible Pavements Division, C. R. R. I, New Delhi-110020

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4. A note on inversion and uniqueness theorems for generalized Mehler-Fock transformation,
P. C. Jain and R. D. Agrawal, S. A. T. I., Vidisha
5. Network Analysis,
D. K. Singhai, S. A. T. I., Vidisha, M. P.
6. A maximum entropy analysis for $M^x/G/1$ queueing system of equilibrium, Madhu Jain. D. A. V. (P. G.) College, Dehradun
7. Fractional derivatives of confluent hypergeometric froms of Karlsson's multiple hypergeometric function ${}^{(k)}F_{CD}{}^{(n)}$,
R. C. Singh Chandel and P. K. Vishwakarma
D. V. Postgraduate College, Orai-285001, U. P.
8. A theorem on system of differential equations,
S. N. Singh and Chadrama Singh, B. D. College, Bihiya, Bihar
9. Multiple hypergeometric function of Srivastava and Daoust and its applications in two boundary value problems,
R. C. Singh Chandel and Abha Tiwari,
D. V. Postgraduate College, Orai-285001, U. P.
10. Unified study of two general classes of functions.
R. C. Singh Chandel and Shashi Agrawal,
D. V. Postgraduate College, Orai-285001, U. P.
11. Hagen-Poiseuille flow of suspension in liquid,
D. K. Wagh, Shri G. S. I. T. S, Indore, M. P.
12. Some expansions for the H-function of several variables with series involving Jacobi polynomials,
A. K. Ranghe, S. S. L. Jain School, Vidisha, M. P.
13. Goal programming approach in motivation process,
D. Nag, Statistician, Govt. Hospital, Vidisha
14. Correlation filters for optical pattern recognition,
R. P. Srivastava C/o Dr. A. K. Tiwari, M. A. C. T, Bhapal.

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15. Few applications of mathematics in power system engineering,
D. K. Singhai, Reader in Electrical Engineering, S. A. T.I., Vidisha
16. Some results for the maps with non-unique fixed point,
Arvind Bohare and Pratima Rawat, Sagar University, Sagar
17. Some methods of construction of PBIB design,
Ramesh Chand Khot, M. V. M., Bhopal, M. P.
१८. बहुमानी प्रतिचित्रणों के पुनरावृत्तिकों का अभिसरण,
S. L. Singh and U. C. Gairola, G. K. University, Hardwar
19. Fixed point theorem for theree densifying mappings in bounded Complete metric space,
R. K. Pandey, Govt. P. G. College, Narsinghpur-487001, M. P.
20. Unique fixed point theorem in Hausdorff space,
Anil Rajput c/o Dr. R. S. Chandel,
Govt. Science Commerce College, Benazir, Bhopal, M. P.
21. A note on the degree of approxmation to continuous functions,
Manoj Singh, Govt. Polytechnic, Ujjain-456010, M. P.

Sixth Technical Session: (1 .00 noon)

SYMPOSIUM CONT.

Chairperson: Dr. P. M. Gupta
Ex-Professor and Head of Mathematics Department,
S. A. T. I., Vidisha

Talk 1. Vedic Mathematics for Engineers by Dr. Narendra Puri

Chairperson: Dr. J. M. C. Joshi, Prof. and Head
Kumaun University, Nainital

Talk 2. "Springs Saviour and Philosophy of Hermite Polynomials" by Dr P. K. Subramanian, Professor of Nuclear Physics, Madras University, Madras

Chairperson: Dr. Narendra Puri Reader, Civil Engg. Deptt.
University of Roorkee, Roorkee

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Talk 3. **"ABACUS"**, by Dr. R. P. Singh,
State Field Adviser, N. C. E. R. T., R. C. E., Bhopal

TEA-BREAK

Meetings of General Body and Executive Council: (1.30 P. M.)

VALEDICTORY FUNCTION: (2.30 P. M.)

Chief Guest: Prof. H. N. Silakari, Principal,
S. A. T. I. (Engg. College) Vidisha, M. P.

Chairperson: Dr. P. M. Gupta
Ex-Professor and Head, Mathematics Department,
S. A. T. I., Vidisha

LUNCH (3.30 P. M.)

VISIT TO SANCHI: (4.15 P. M.)

DINNER (8.00 P. M.)

LIST OF PARTICIPANTS

1. Shri Raghavji, M. P., Vidisha (Chairman)
2. Shri M. P. Shukla, State Man Power Planning Minister, Madhya Pradesh (Chief Guest)
3. Shri Chhabi Ram Argal, M. P., Morana } (Special Guests)
4. Shri Rajendra Sharma, M. P. } (Special Guests)
5. Professor H. N. Silakari, Principal, S. A. T. I., Vidisha
6. Professor M. K. Jain, Vice-Chancellor, A. P. University, Rewa
7. Dr. Narendra Puri, Civil Engineering Department, Roorkee University, Roorkee, U. P.
8. Professor P. L. Sharma, Sagar University, Sagar, M. P.
9. Professor V. P. Saxena, Jiwaji University, Gwalior, M. P.
10. Dr. D. S. Karaulia, T. T. T. I., Shyamla Hills, Bhopal, M. P.
11. Professor G. C. Sharma, Agra Uaiversity, Agra

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12. Professor P. R. Subramanian, Madras University, Madras
13. Professor S. N. Pandey, M. M. M. Fngineering College, Gorakhpur
14. Professor J. M. C. Joshi, Kumaun University, Nainital, U. P.
15. Professor S. L. Singh, G. K. University, Hardwar, U. P.
16. Dr. Sundr Lal, Agra University, Agra
17. Professor R. P. Singh, State Field Adirsor, N. C. E. R. T., R. C. E., Shyamla Hills, Bhopal
18. Professor P. M. Gupta, Ex-Professor and Head of Mathematics Department, S. A. T. I., Vidisha
19. Professor R. D. Agrawal, (Local Secretary) S. A. T. I., Vidisha
20. Dr. R. C. Singh Chandel, Secretary, Vijnana Parishad of India, D. V. Postgraduate College, Orai-285001, U. P.
21. Mr. Arvind Bohare, Sagar University, Sagar
22. Mr. Hemant Kumar, C/o-Department of Mathematics, D. V. Postgraduate College, Orai-285001, U. P.
23. Dr. S. K. Ghosh, Flexible Pavements Division, C. R. R. I, New Delhi-110020
24. Mr. P. C. Jain, S. A. Tech. Institute, Vidisha, M. P.
25. Mr. D. K. Singhai S. A. T. I., Vidisha M. P.
26. Dr. (Mrs.)Madhu Jain, D. A. V. (P. G) College, Dehradun, U. P.
27. Dr. Ramesh Chand Khot, M. V. M., Bhopal
28. Mr. Umesh Chand Gairola, G. K. University, Hardwar
29. Mr. P. K. Vishwakarma, C/o-Department of Mathematics, D. V. Postgraduate Collcge Orai-285001, U. P.
30. Dr. S. N. Singh, B. D. College, Bihiya, Bihar
31. Ms. Abha Tiwari, C/o-Department of Mathematics, D. V. Postgraduate College, Orai
32. Dr. A. B. Mathur, Delhi College of Engineering, Delhi
33. Dr. S. K. Bhargava, S. P. I. C., Jhani, U. P.

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34. Dr. A. K. Gupta, 98-Gopalganj, Navin Printing Press, Orai, U. P.
35. Mr. R. K. Pandey, Govt. P. G. College, Narsinghpur-487001, M.P.
36. Mr. Anil Rajput, C/o-Dr. R. S. Chandel, Govt. Science and Commerce College, Benazir, Bhopal, M. P.
37. Mr. A. K. Ranghe, Lecturer, S. S. L. Jain School, Vidisha
38. Mr. D Nag, Statistician, Govt. Hospital, Vidisha, M. P.
39. Ms. S. Agrawal. C/o-Department of Mathematics, D. V. Postgraduate College, Orai-285001, U. P.
40. Mr. R. P. Srivastava, C/o-Dr. A. K. Tiwari, M. A. C. T., Bhopal
41. Mr. Manoj Singh, Govt. Polytechnic, Ujjain-456010, M. P.
42. Dr. D. K. Bagh, Shri G. S. I. T. S., Indore, M. P.
43. Mr. D. K. Singhai, Electrical Engineering Department, S. A. T. I., Vidisha.
44. Dr. Raghvendra Singh Chandel, Govt. Science and Commerce College, Benajir, Bhopal, M. P.
45. Dr. D. C. Shukla, Principal, Higher Sccondary School, Mobai (Tikamgarh), M. P.
46. Dr. C. K. Sharma, A. P. University, Rewa, M. P.
47. Dr A. V. Dange, S. S. L. Jain College, Vidisha
48. Mr. Divya Kumar Bagga, Barktullah University, Bhopal
49. Mr. K. K. Kumbhare, Govt. Degree College, Ganj Basoda, M. P.
50. Mr. S. K. Nigam, A. P. University, Rewa M. P.
51. Mr. A. K. Dixit, Lecturer, Govt. Higher Secondary School, Karmgarh, M. P.
52. Mr. Anil Soni, Lecturer, Govt. Higher Secondary School, Gulabganj, M. P.
53. Miss Pratima Rawat, University of Sagar, Sagar, M. P.
54. Miss Nilima Sharma, Sagar University, Sagar
55. Miss Pratibha Arya, Govt. Girls College, Vidisha
56. Miss Pavika Shukla, G-8, Sadar Colony, Narsinghpur-487001, M.P.
57. D. P. Shukla, S. A. T. I., Vidisha

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58. Dr. R C. Jain, Department of Mathematics, S. A. T. I., Vidisha
59. Dr. D. P. Goyal, S. A. T. I., Vidisha
60. Professor Rajendra Dubey, S. A T. I., Vidisha
61. Professor Aail Goyal, S. A. T. I., Vidisha
62. Professor A. G. K. Murthy, S. A. T I., Vidisha
63. Professor D. S. Joshi, S. A. T. I., Vidisha
64. Professor P. K. Markedkar, S A. T I., Vidisha
65. Professor J. C. Gupta, S. A. T. I., Vidisha
66. Professor S N. Agrawal, S A. T. I., Vidisha
67. Professor A. S. Mehta, S. A. T. I., Vidisha
68. Professor R. C. Jain, Department of Applied Mechanics, S. A. T. I.. Vidisha
69. Professor R. P. Vashishtha, Department of Chemistry, S. A. T. I., Vidisha
70. Professor J. K. Jain, S. A. T. I.. Vidisha
71. Professor A. K. Yadava , Reader, Electrical Engineering Department S A. T. I., Vidisha
72. Professor Sandeep Jain, S. A. T. I., Vidisha
73. Professor K. C. Jain, S A. T. I., Vidisha
74. Professor Lokesh Bajpai, S. A. T. I., Vidisha

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